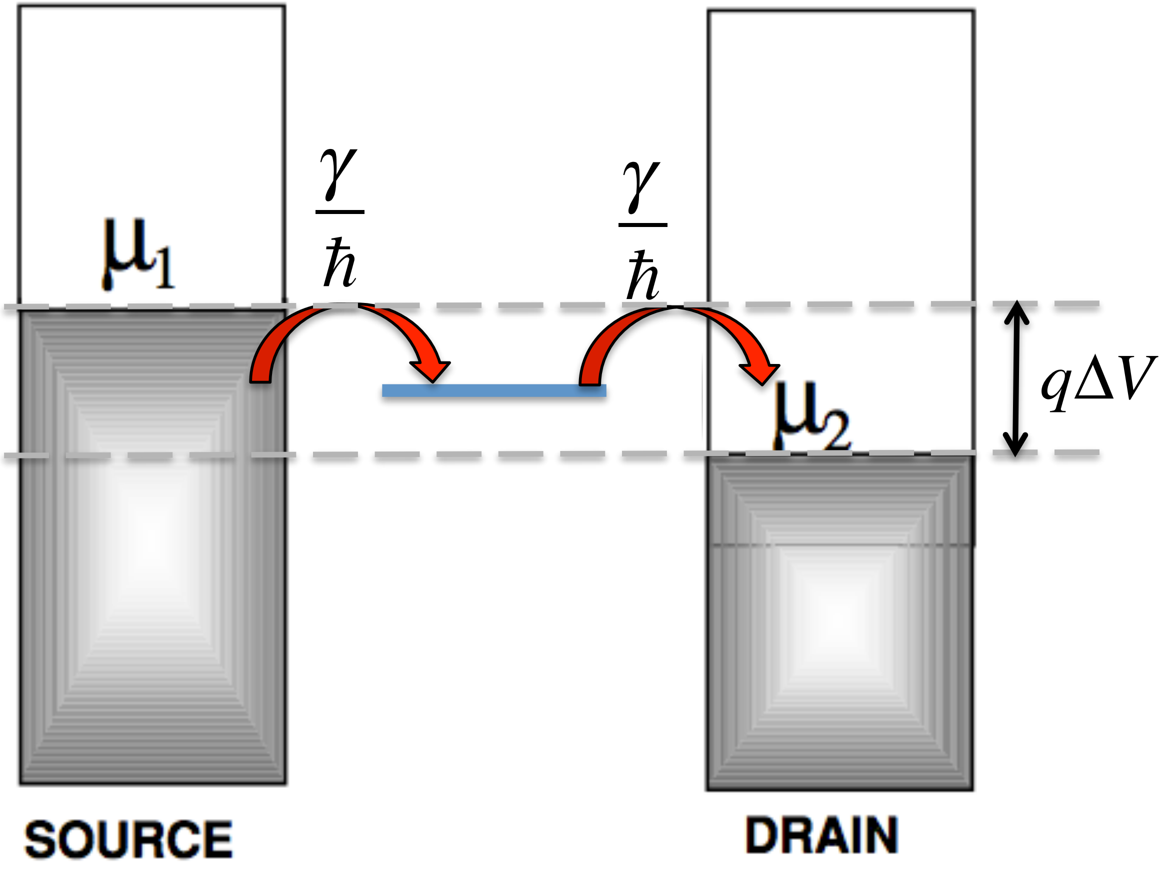
Fundamentals of NanoElectronics

Lecture – II Review

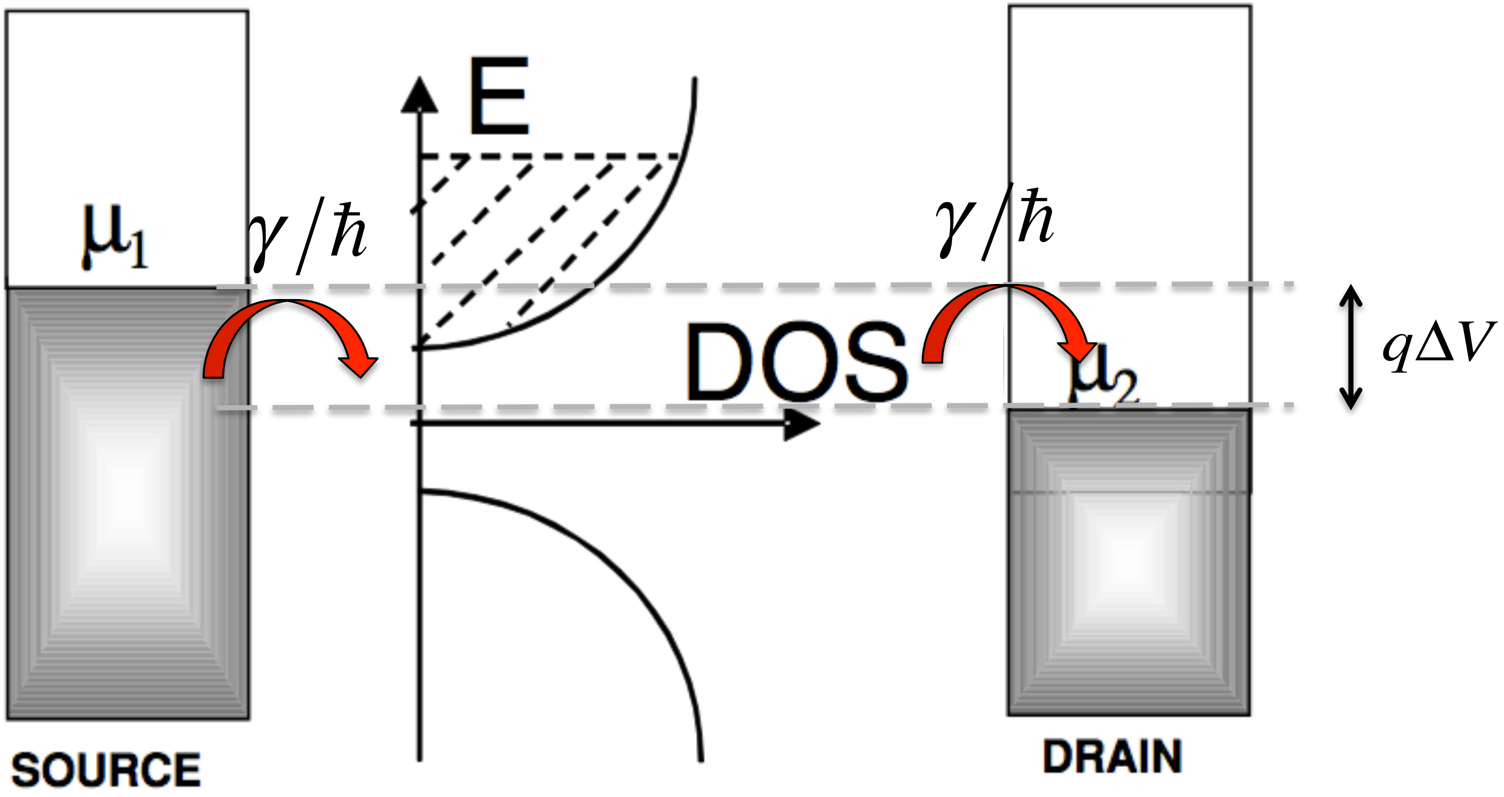
SINGLE LEVEL TRANSPORT (T→0)



Applied Voltage

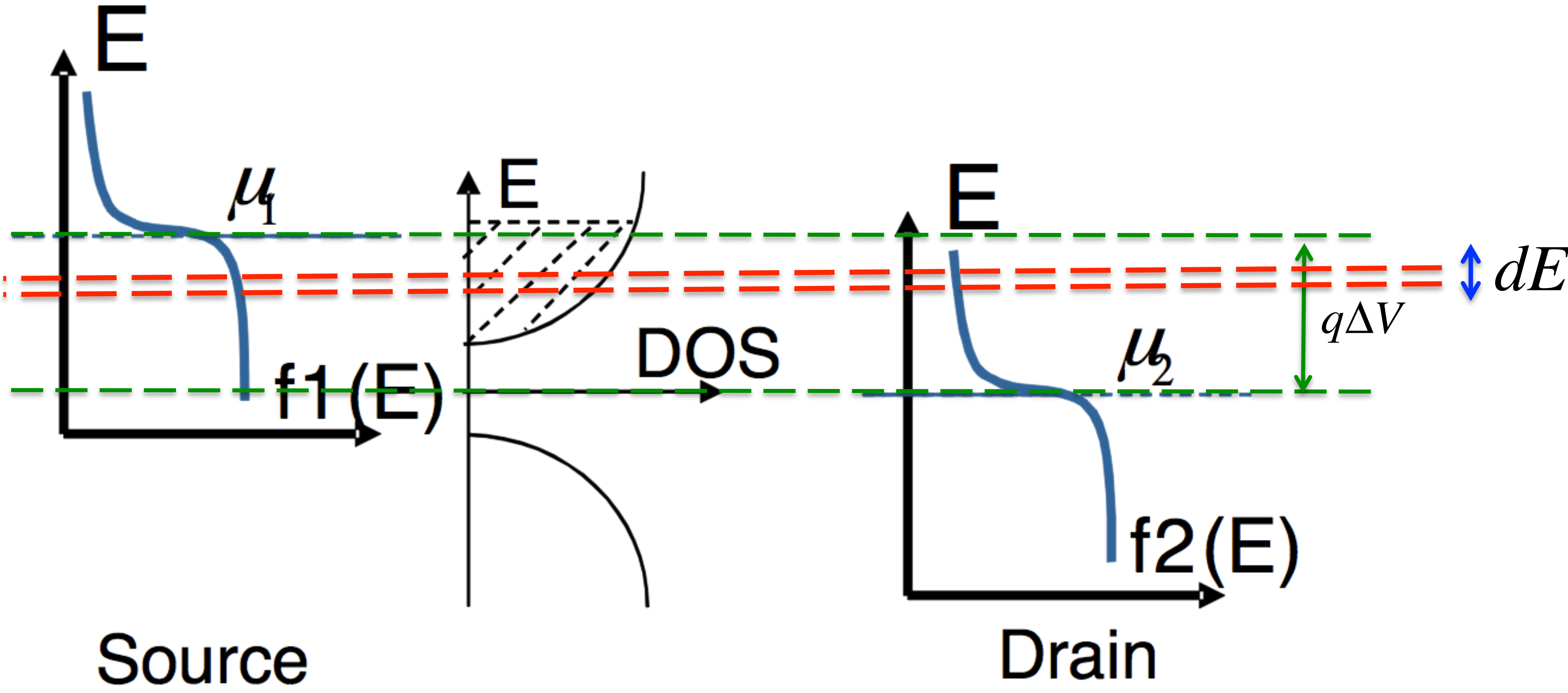
Current: $I = \frac{\Delta Q}{\Delta t} = \frac{q\gamma}{2\hbar}$ $\Delta t = \frac{2\hbar}{\gamma}$

MULTIPLE STATES (T→0)



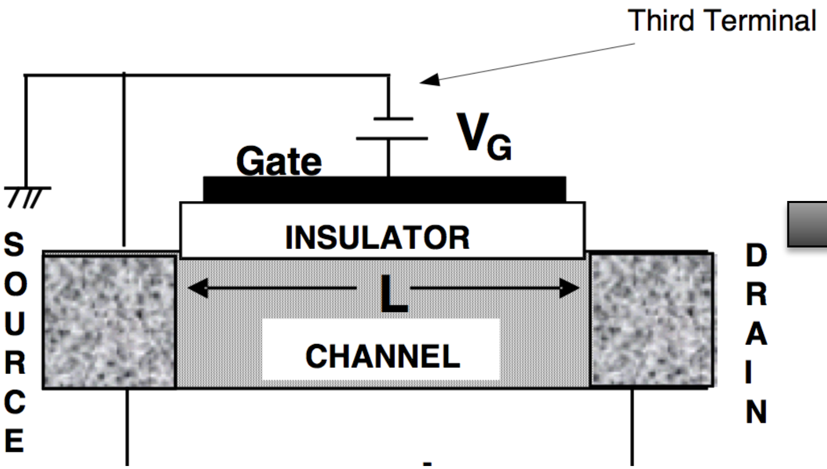
Current : $I = \frac{q\gamma}{2\hbar}\int dE\ D(E)$

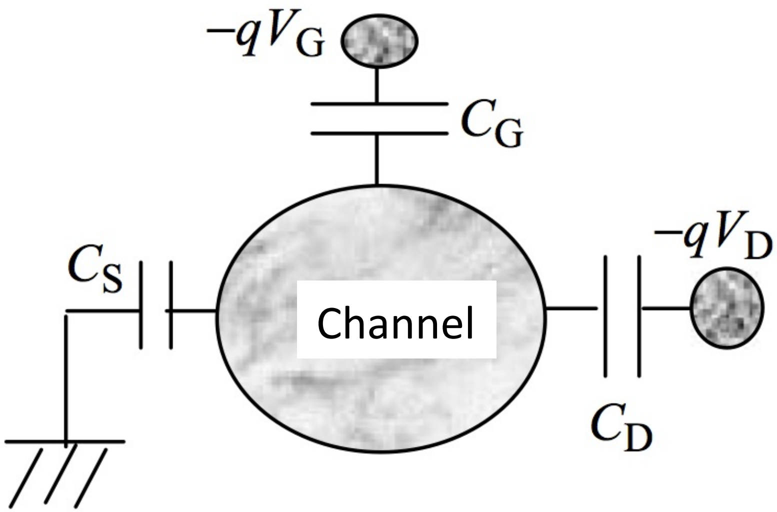
CURRENT MORE GENERAL FORM (T≠0)

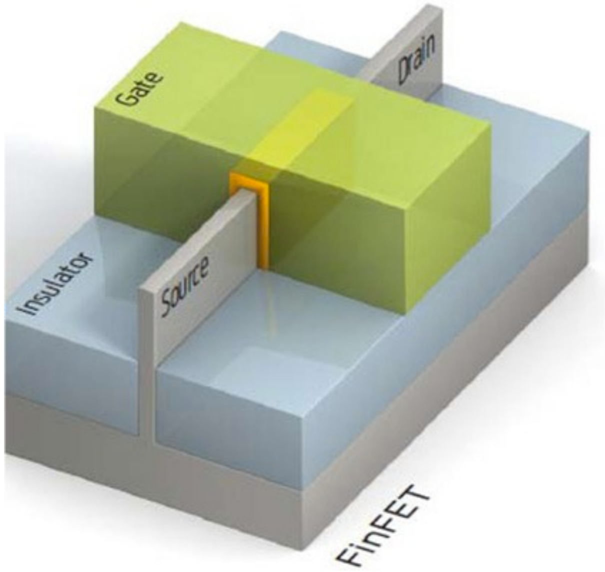


Current : $I = \frac{q\gamma}{2\hbar}\int dE\ D(E)\left[f_1(E) - f_2(E)\right]$

Gate Control (FET TRANSISTOR)

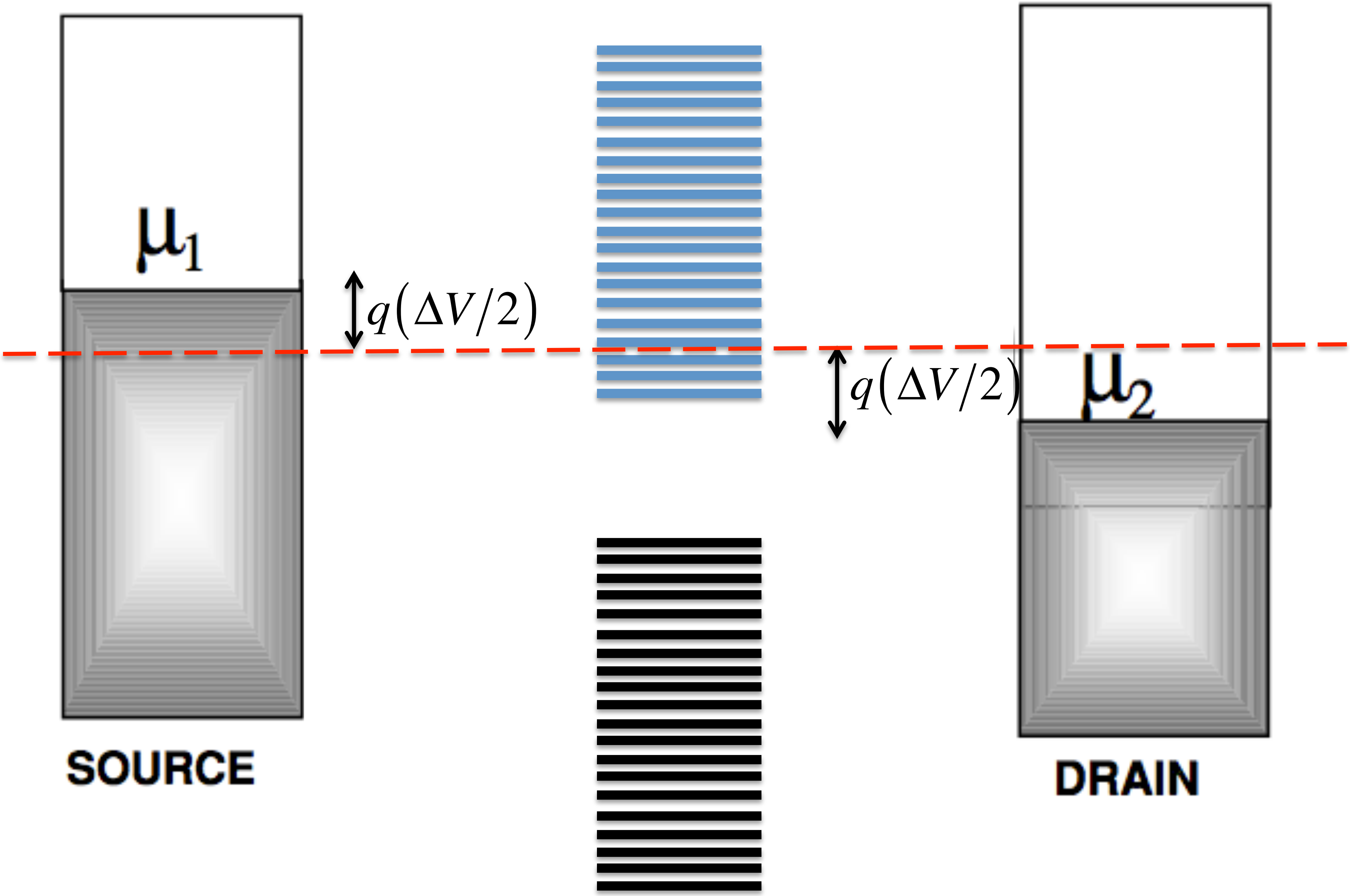




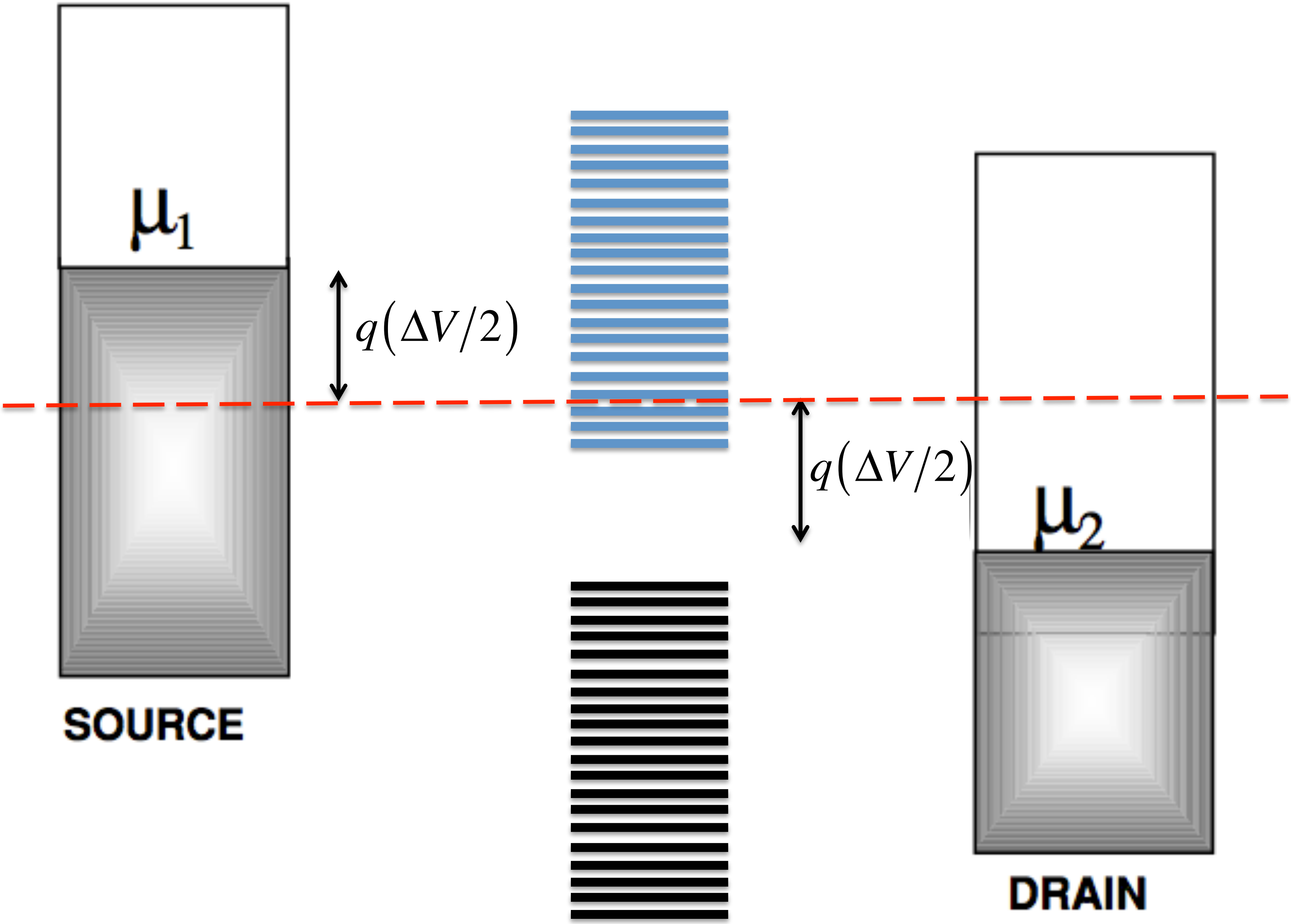


$$\boxed{C_G = \frac{\varepsilon A}{d}}$$

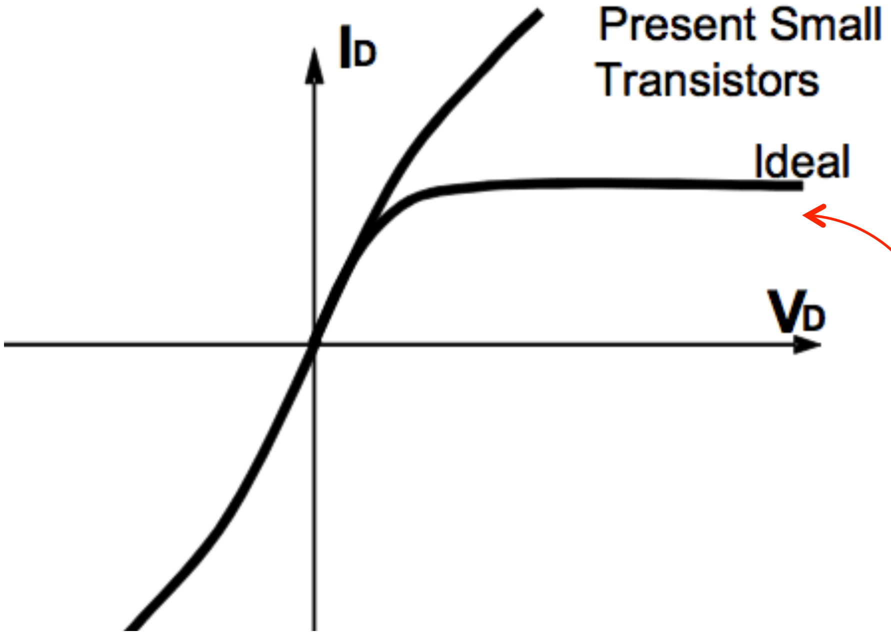
VOLTAGE DROP



VOLTAGE DROP



CURRENT SATURATION??

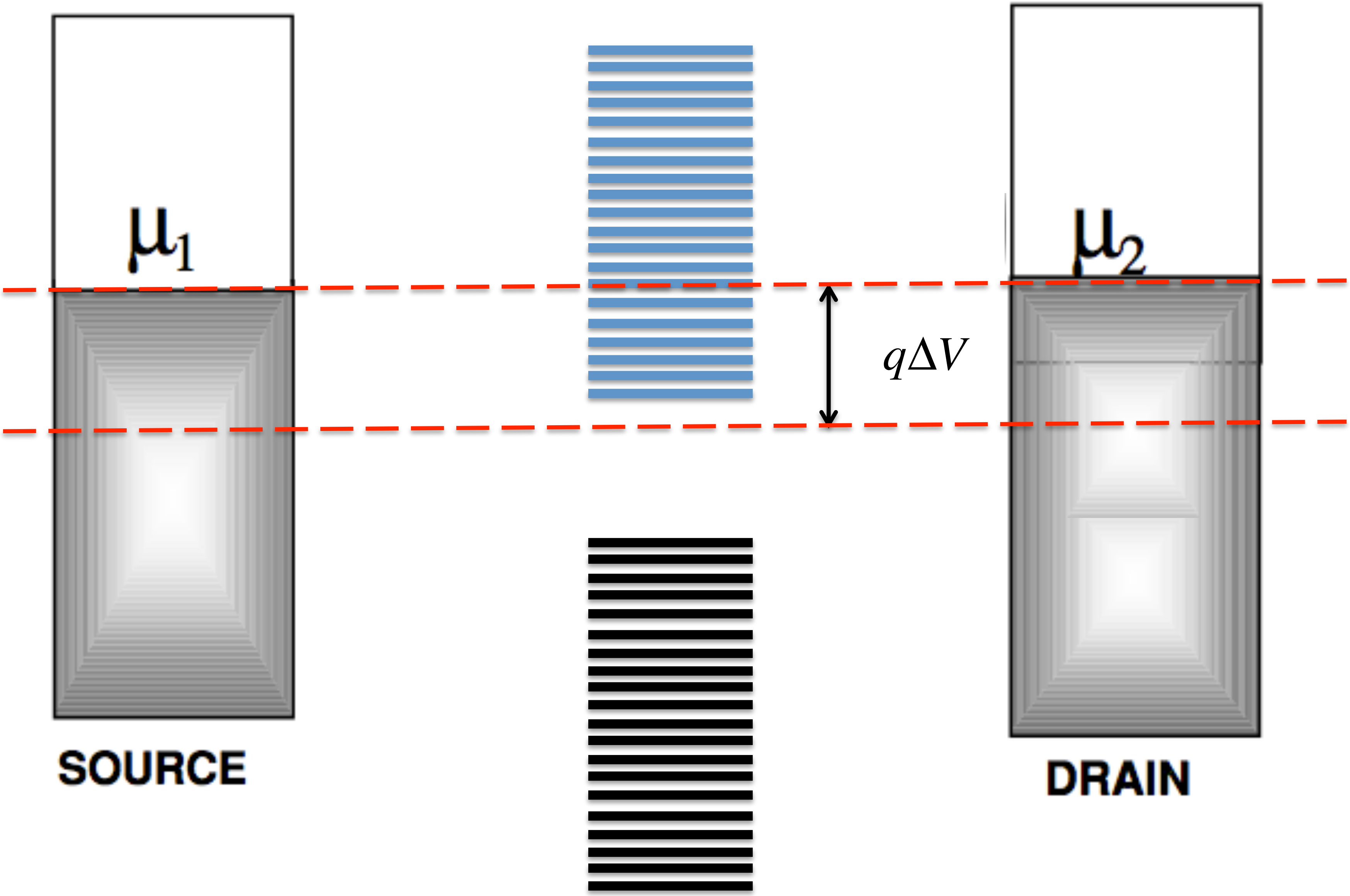


What is the channel potential?

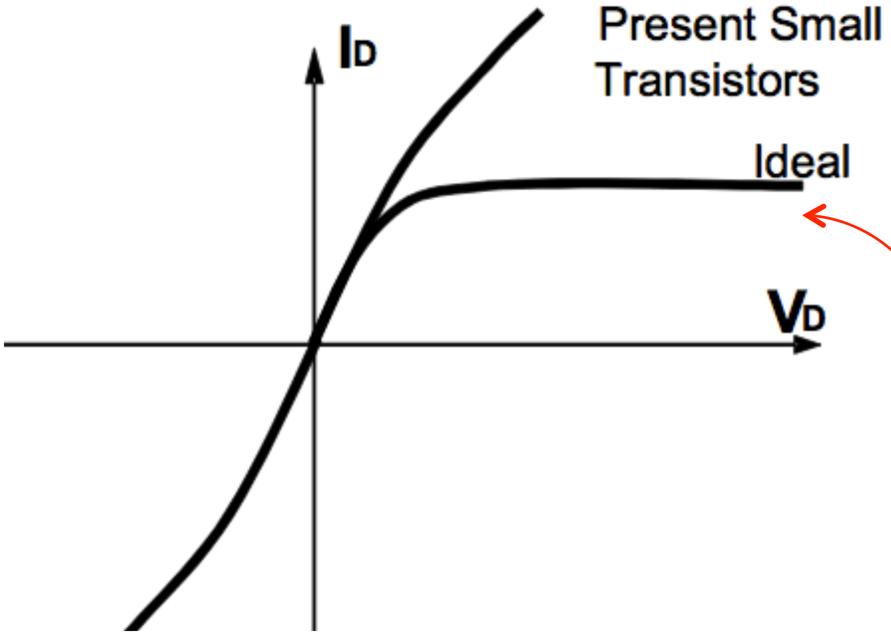
How is that possible?

Saturation is ideal for circuit design

APPLIED BIAS



CURRENT SATURATION??



What is the channel potential?

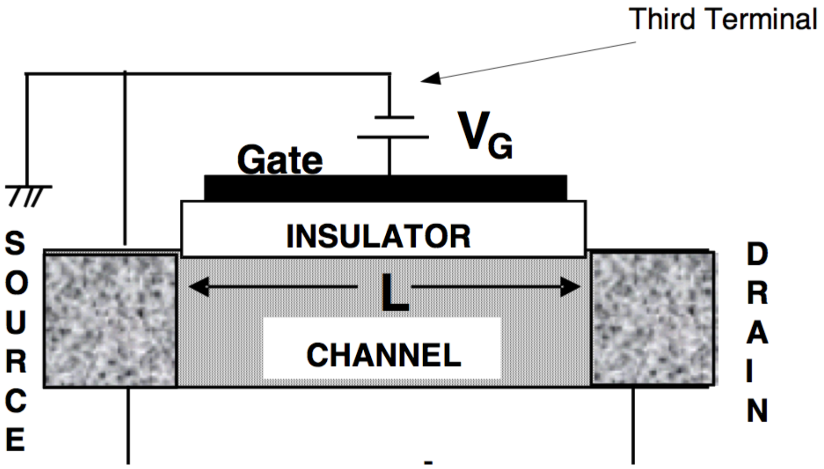
How is that possible?

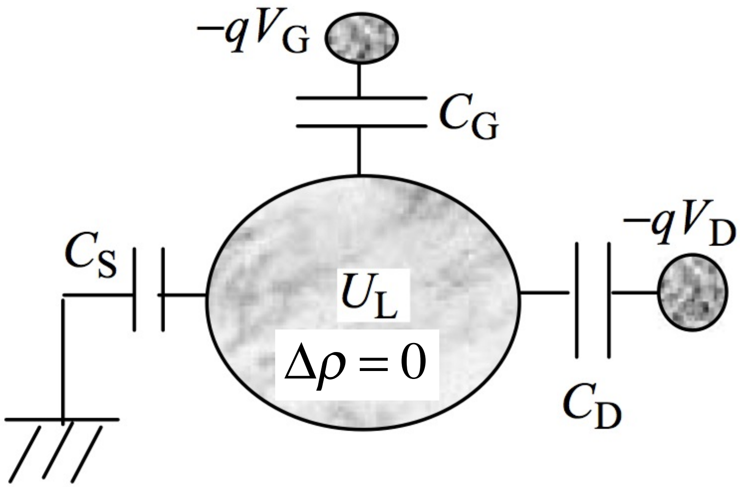
Saturation is ideal for circuit design

Fundamentals of NanoElectronics

Lecture – III

Capacitor Network (Laplace Potential)





$$\vec{\nabla}\cdot\left(\varepsilon_r \vec{\nabla} V\right) = 0$$

Laplace Potential

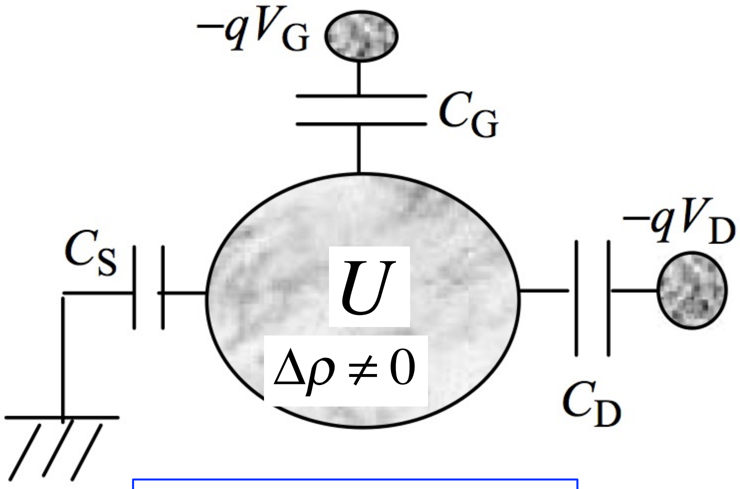
$$\boxed{U_L = \frac{C_S}{C_E}\left(-qV_S\right) + \frac{C_G}{C_E}\left(-qV_G\right) + \frac{C_D}{C_E}\left(-qV_D\right)}$$

where $C_E = C_S + C_G + C_D$

$$\boxed{U_L = \frac{C_G}{C_E}\left(-qV_G\right) + \frac{C_D}{C_E}\left(-qV_D\right)}$$

since $V_S = 0$

Charge Transfer & Charging Potential



$$\boxed{\vec{\nabla}\cdot\left(\varepsilon_r \vec{\nabla} V\right) = -\Delta\rho \big/ \varepsilon_0}$$

Total Charge

$$-q\Delta N = C_S V + C_G\left(V - V_G\right) + C_D\left(V - V_D\right)$$

where $-qV = U$

$$\boxed{U_L = \frac{C_G}{C_E}\left(-qV_G\right) + \frac{C_D}{C_E}\left(-qV_D\right)}$$

$$\boxed{U = U_L + \frac{q^2}{C_E}\Delta N}$$

$$\boxed{U_0 = \frac{q^2}{C_E}}$$

$$\boxed{U = U_L + U_0\left(N - N_0\right)}$$

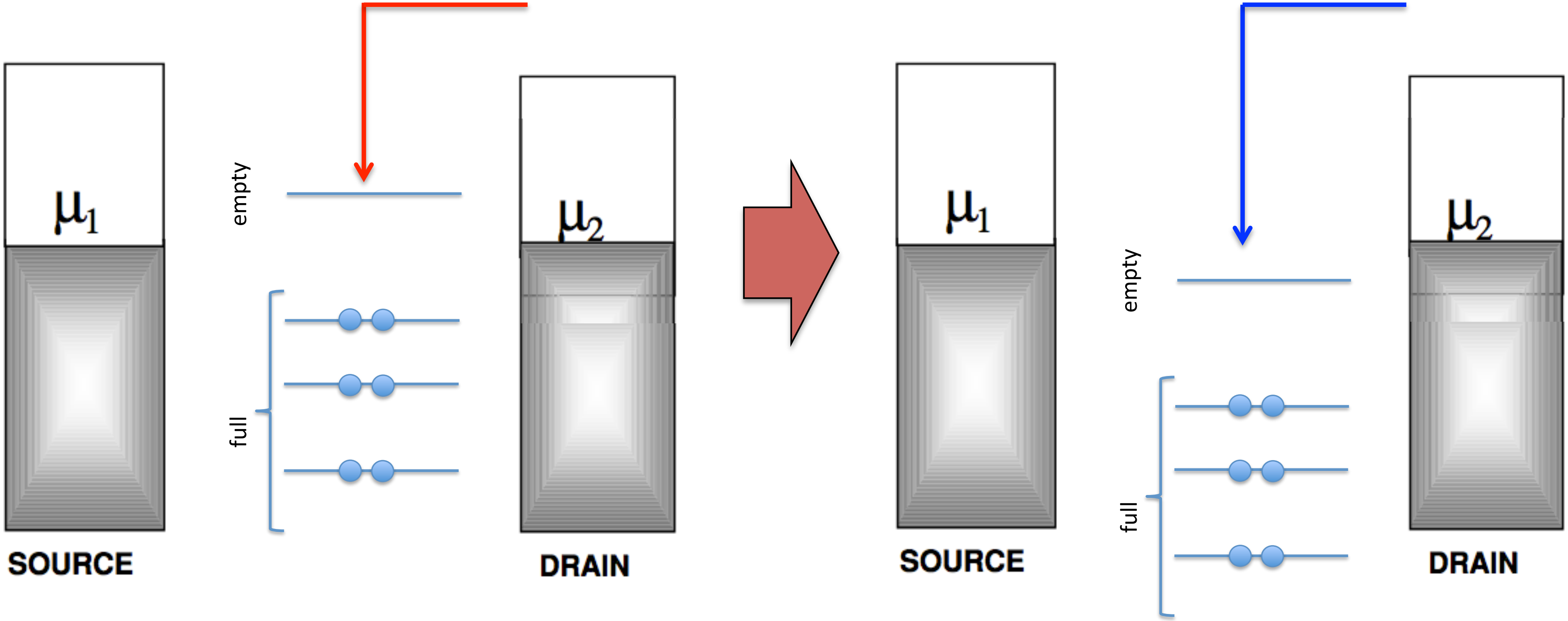
single electron charging energy

STRONG CHARGING REGIME ($U_0 >> kT$)

Coulomb Blockade Effect-I

Two degenerate (same energy)
empty quantum states
(one state for each electron spin direction)

Two degenerate energy levels
are pulled down with gate bias

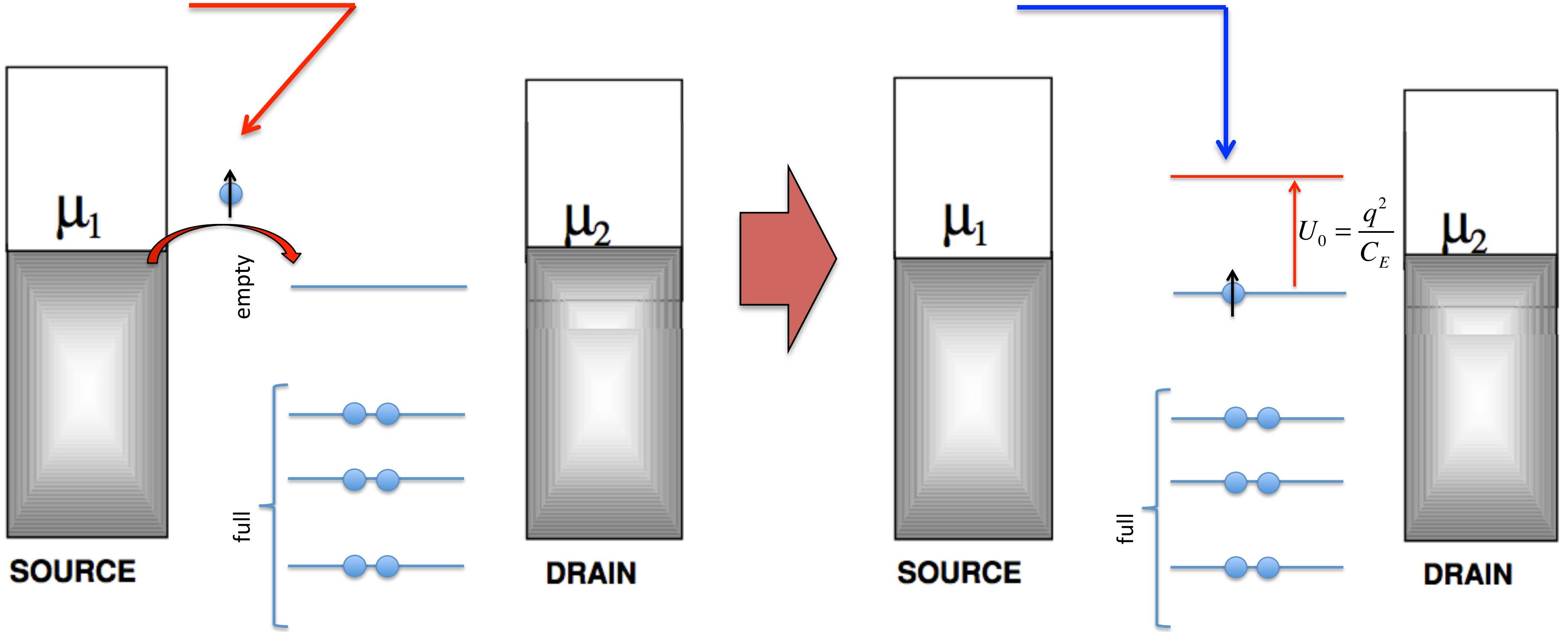


(Equilibrium)

(Gate Voltage Applied)

TIME ARROW → Next Slide!!

Coulomb Blockade Effect-II



First electron jumps into the energy level (let's say spin-up electron)

Degeneracy of the two energy levels is broken due to charging potential U_0 (the extra electron).

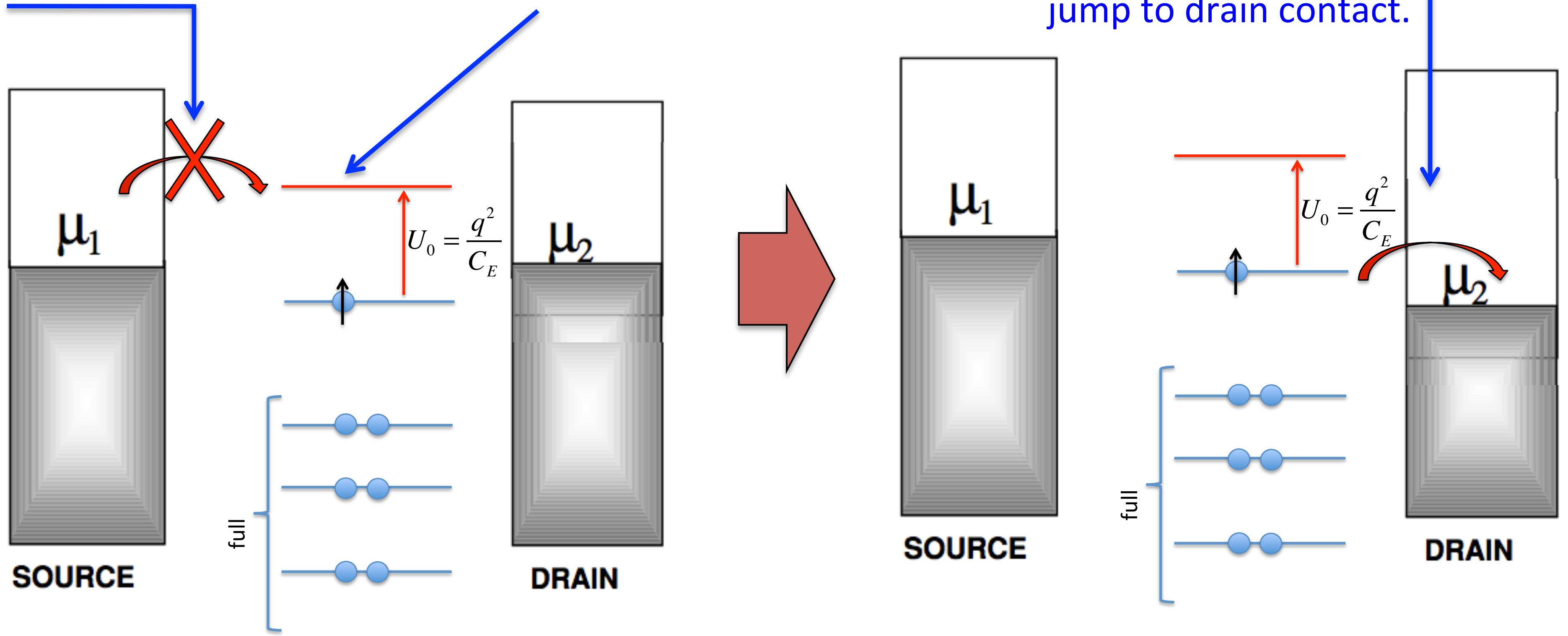
(Gate Voltage Applied)

(Gate Voltage Applied)

TIME ARROW →

Next Slide

Coulomb Blockade Effect-III



Next electron (spin up or down) **cannot** jump In if U_0 is too large!!

Next energy level is too high for contacts to fill in.

If drain bias is applied the electron resting in the channel can jump to drain contact.

(Coulomb Blockade Effect)

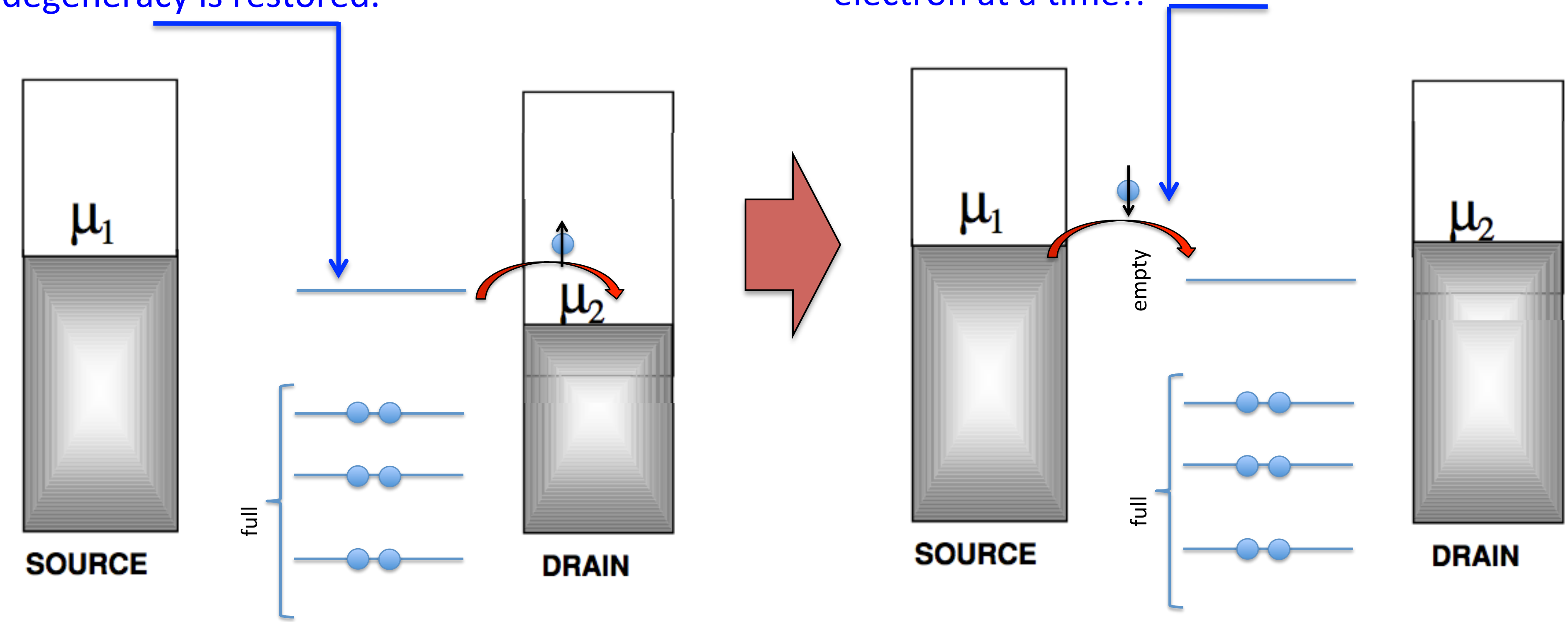
(Bias Voltage Applied)

TIME ARROW → Next Slide

Coulomb Blockade Effect-IV

Once the extra electron leaves the channel, energy level degeneracy is restored.

Now the next electron can jump in from the source. It could be spin-up or spin-down. Process repeats one electron at a time!!

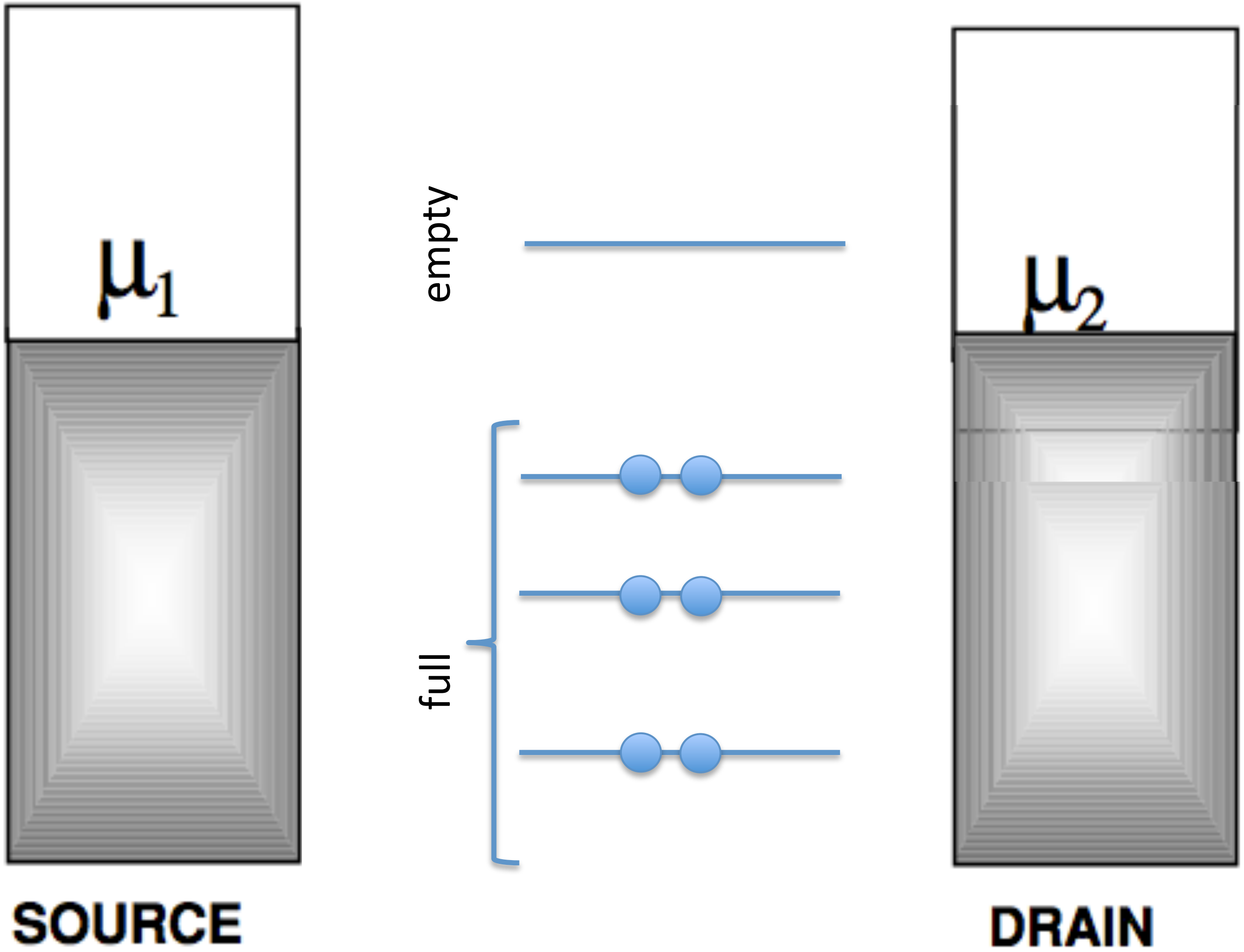


(Degeneracy Restored)

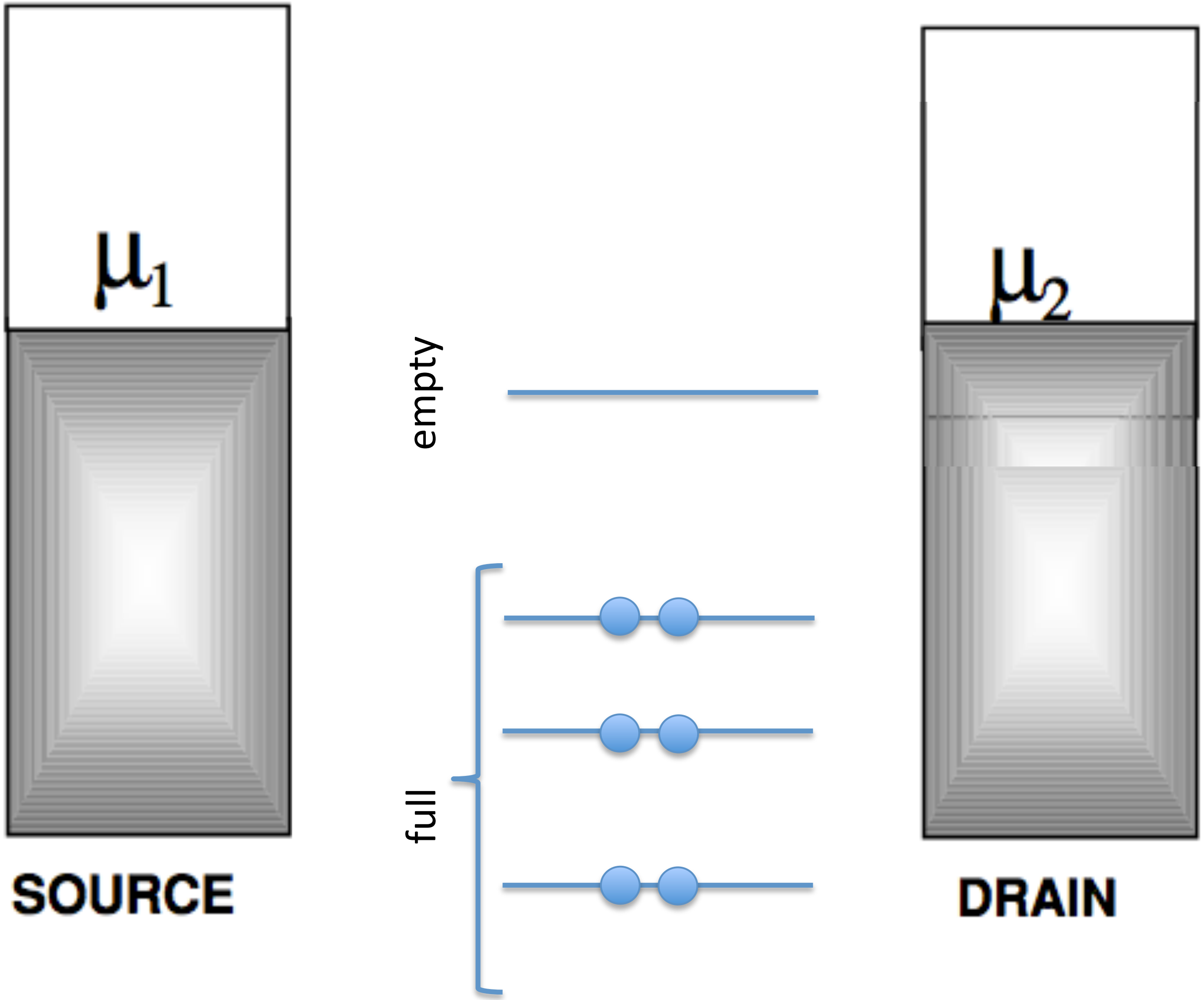
Single Electron Transistor!!!

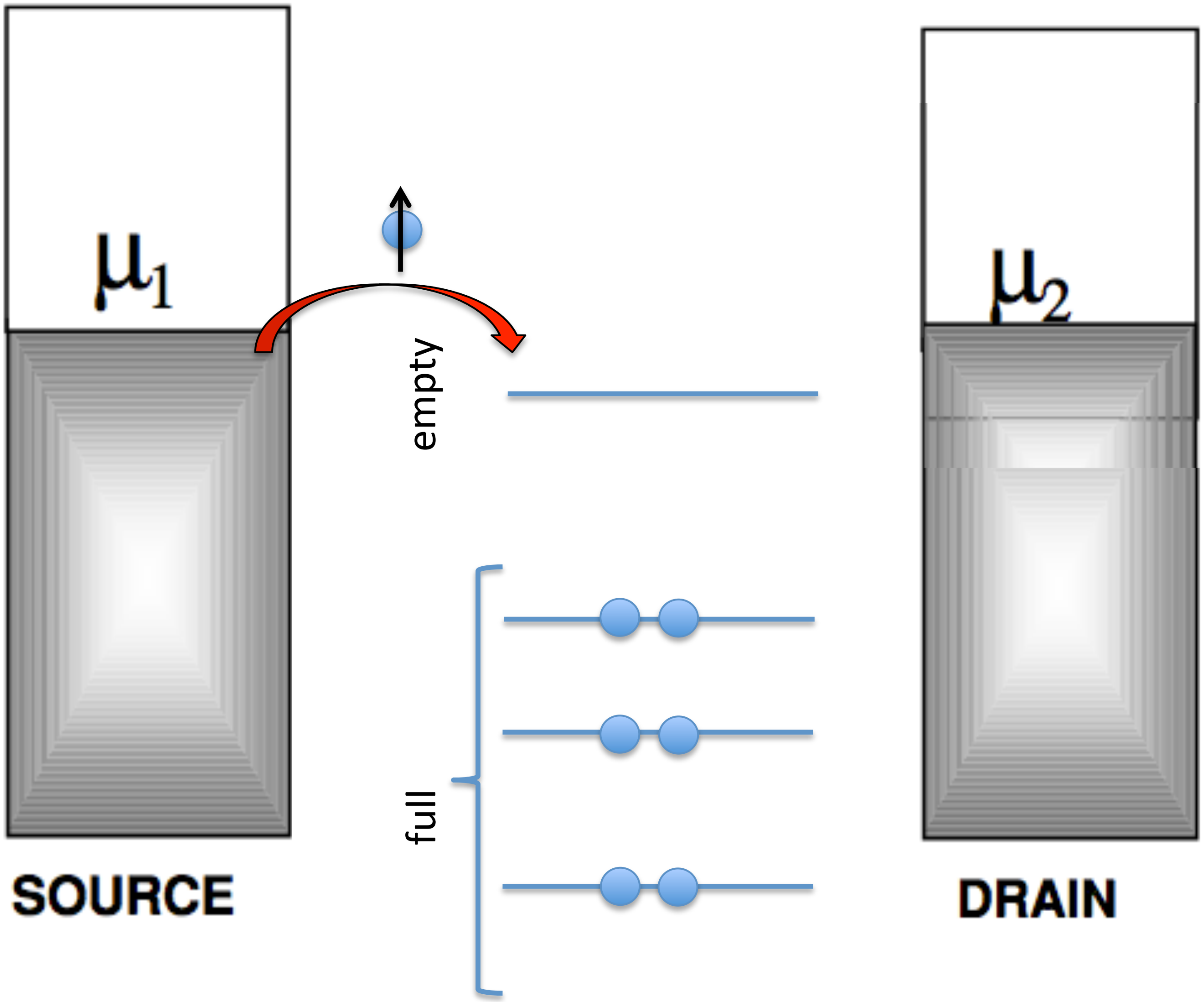
TIME ARROW → Next Electron Please!!

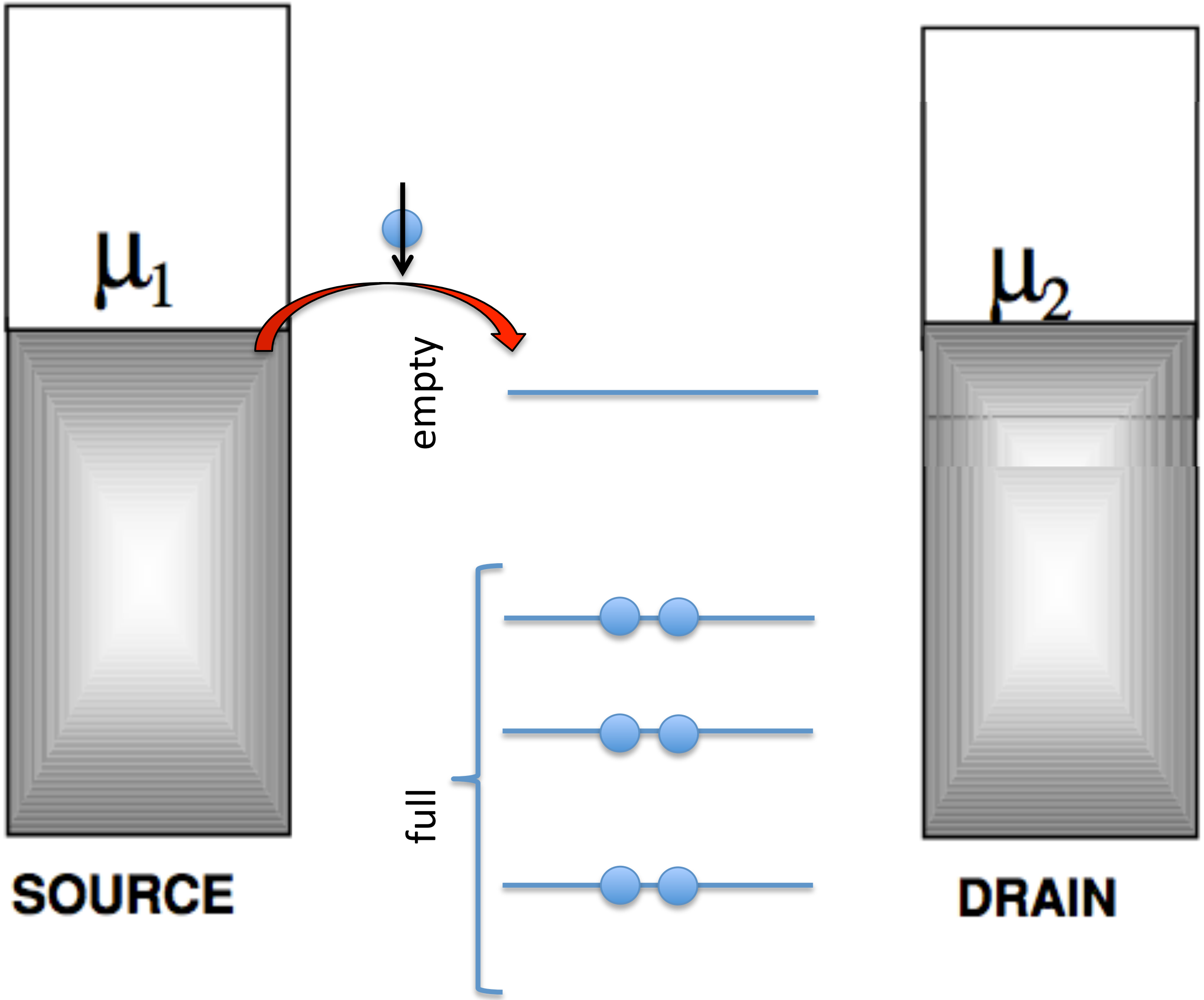
Equilibrium

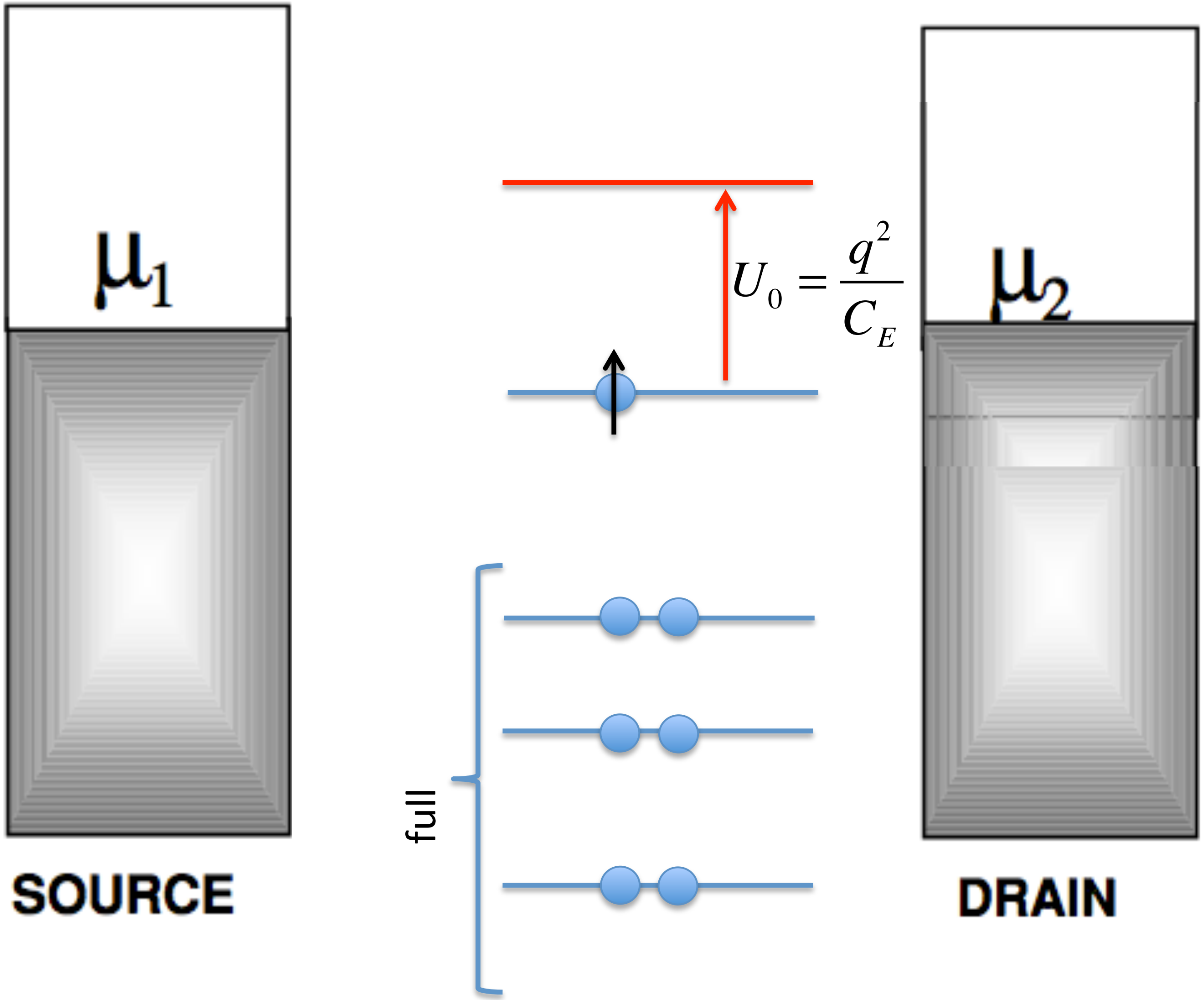


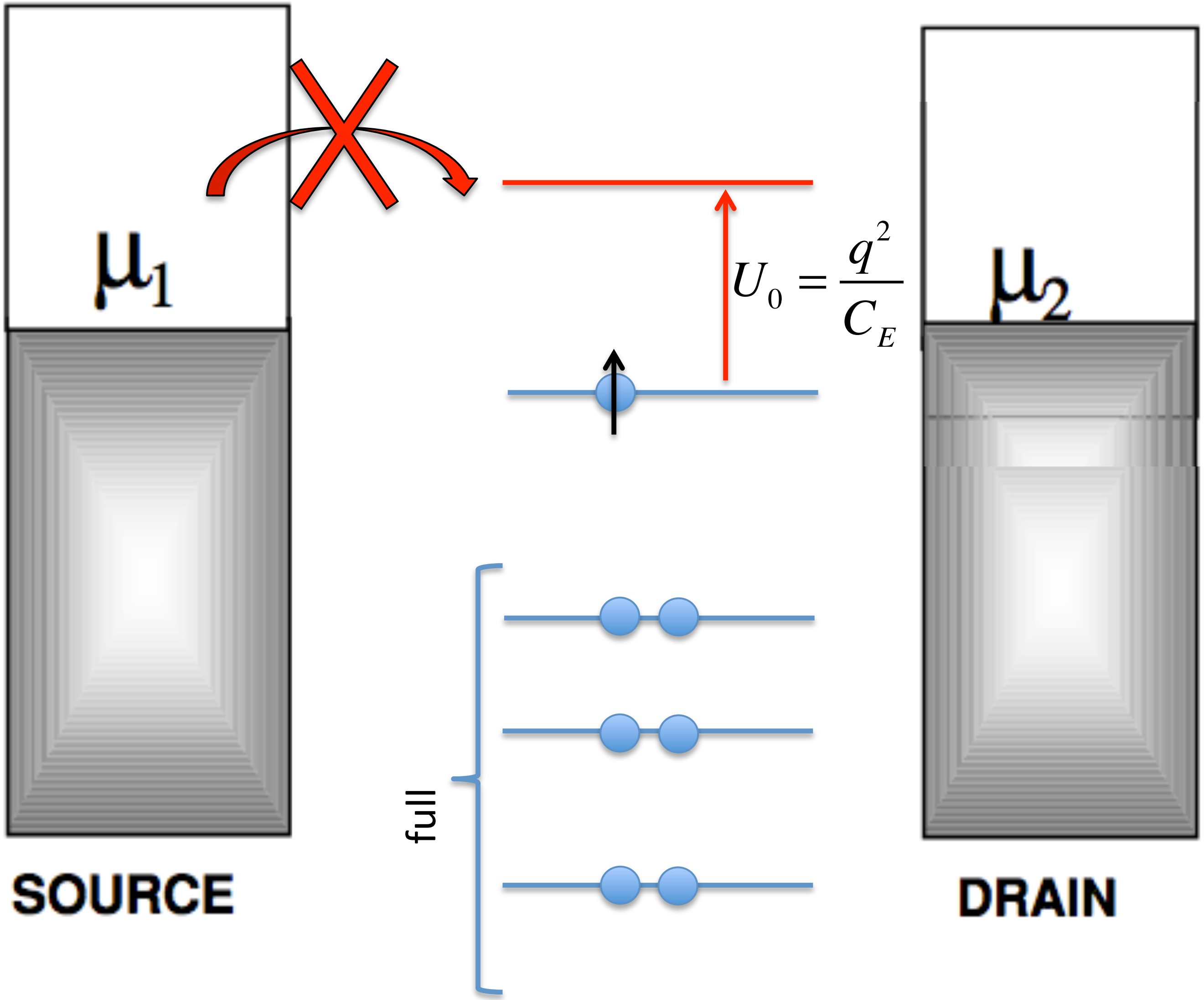
Applied Gate Voltage



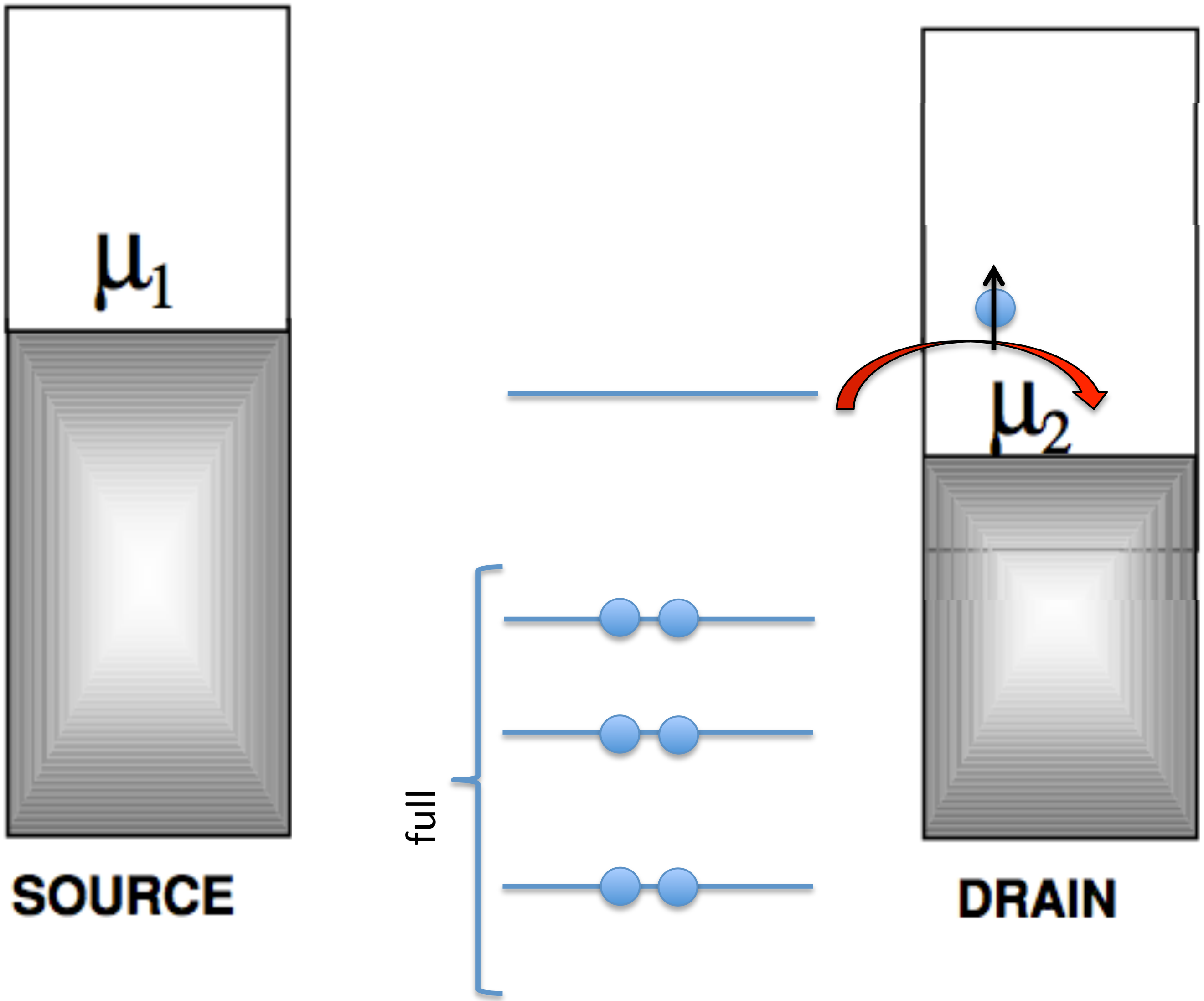

μ_1
μ_2
empty
full
SOURCE
DRAIN


μ_1
μ_2
empty
full
SOURCE
DRAIN


μ_1
$U_0 = \frac{q^2}{C_E}$
μ_2
full
SOURCE
DRAIN

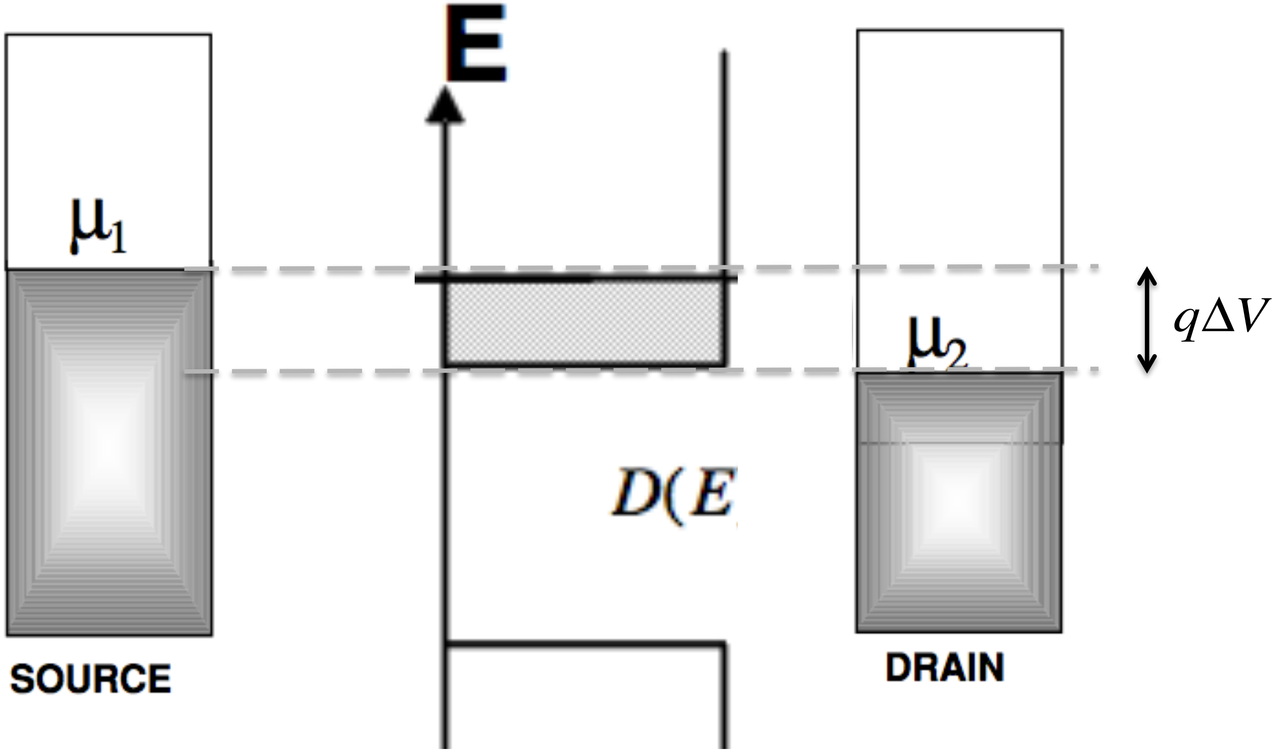


μ_1
$U_0 = \frac{q^2}{C_E}$
μ_2
full
SOURCE
DRAIN


μ_1
μ_2
SOURCE
full
DRAIN

WEAK CHARGING REGIME ($U_0 << kT$)

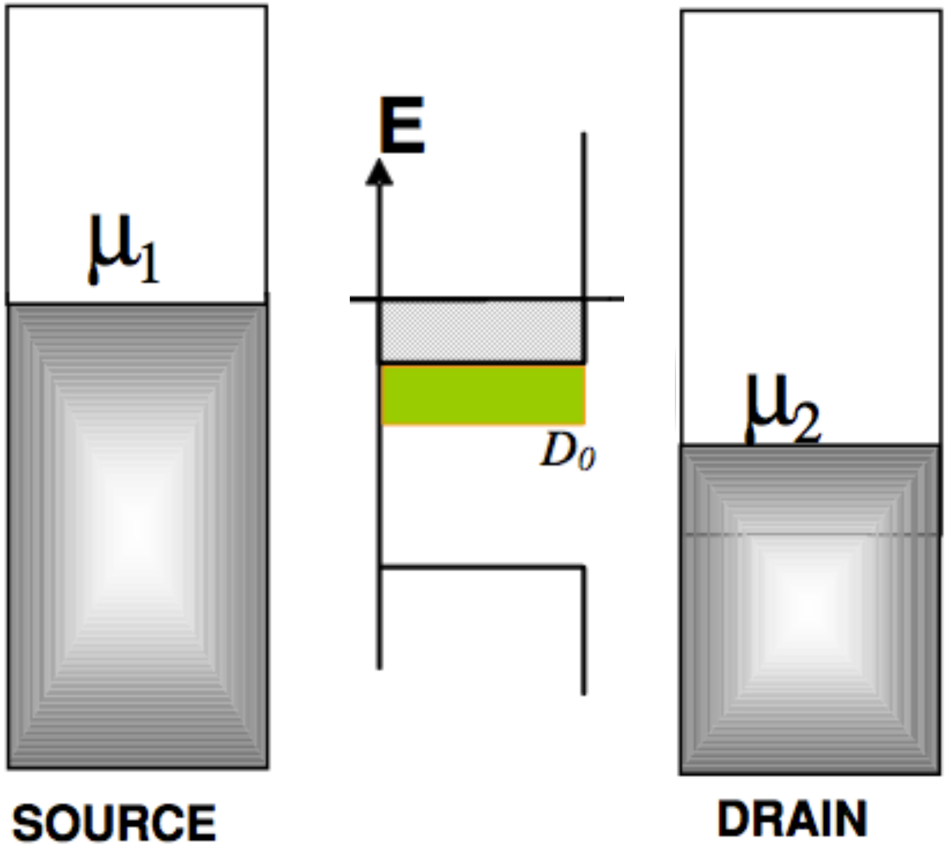
CONSTANT DENSITY OF STATES



Current : $I = \frac{q\gamma}{2\hbar}\int dE\ D(E)\left[f_1(E) - f_2(E)\right] = \frac{q\gamma}{2\hbar}(qVD)$ $\quad\boxed{D(E) = const}$

$$\boxed{I = \frac{q\gamma D}{2\hbar}(q\Delta V)} \Rightarrow \boxed{G = \frac{q^2}{h}(\pi\gamma D)}$$

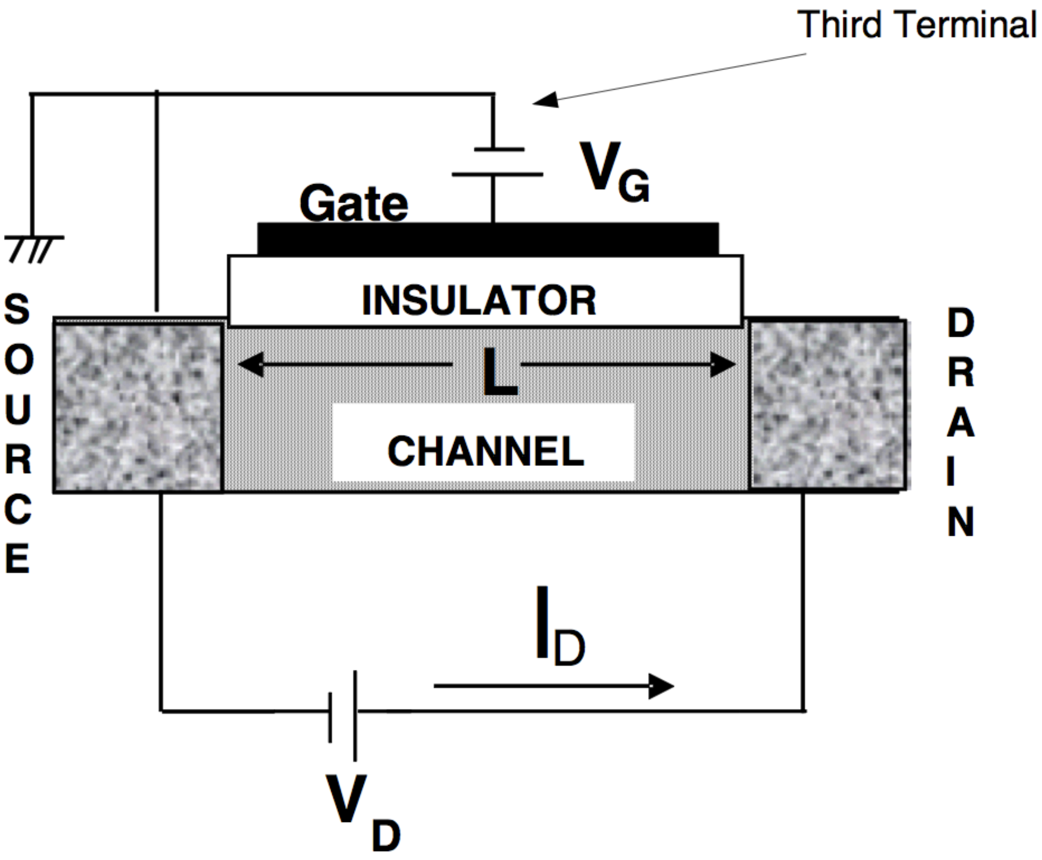
Electron-Electron Interaction

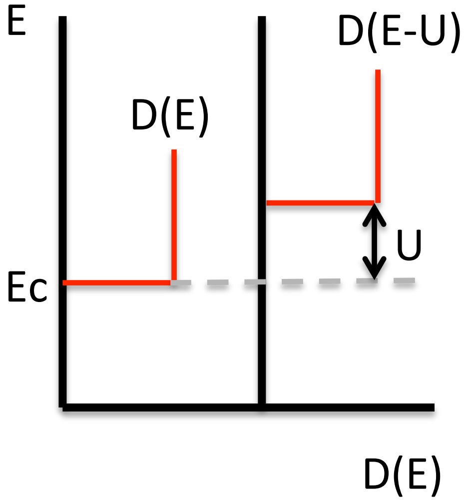


electron-electron interaction →

$$\boxed{U = U_L + U_0 \left(N - N_0 \right)}$$

Gate Control

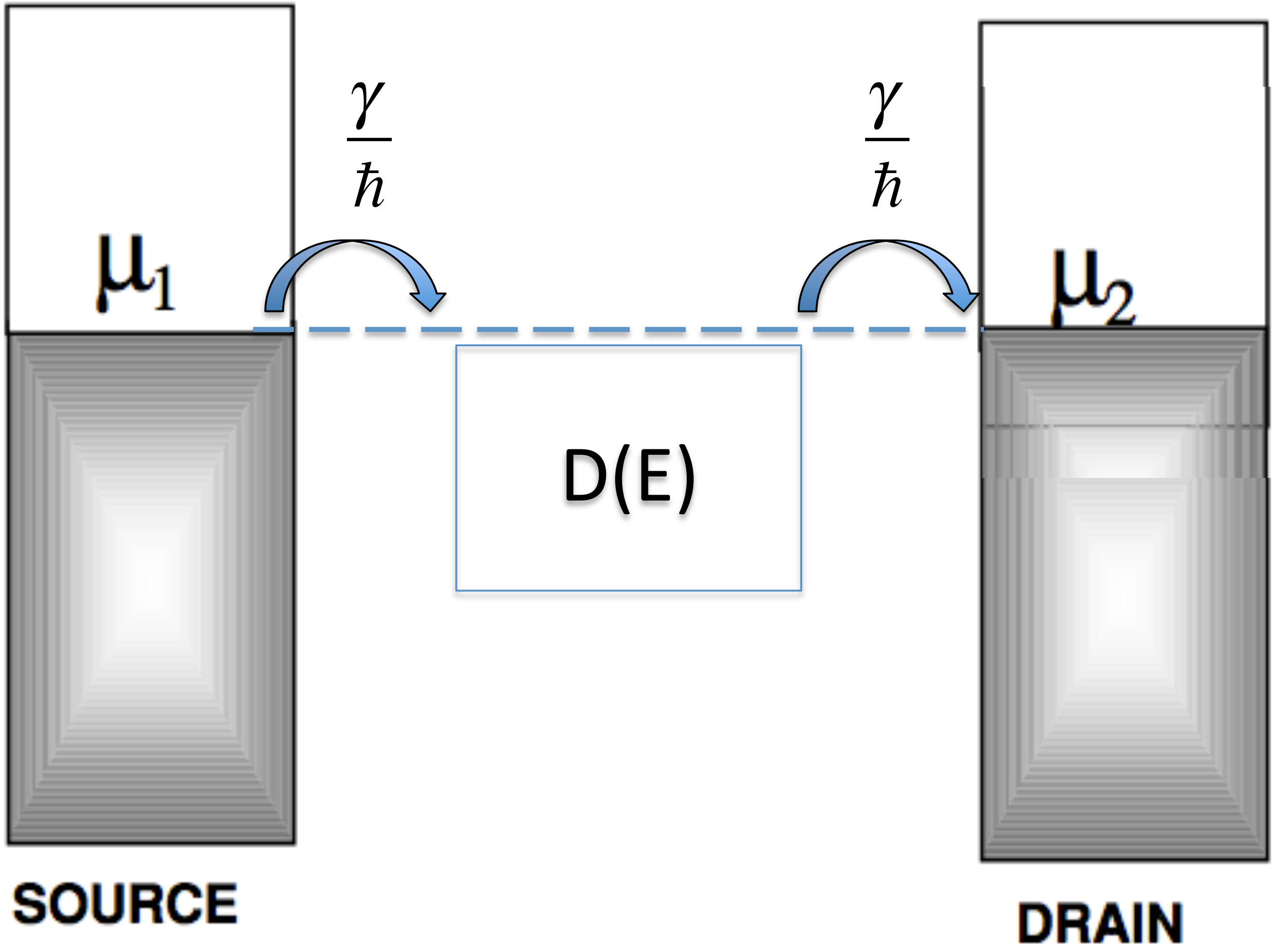




$$I = \frac{q\gamma}{2\hbar}\int_{-\infty}^{\infty} dE \cdot D(E-U)\left[f_1(E) - f_2(E)\right]$$

NUMBER OF ELECTRONS IN THE CHANNEL

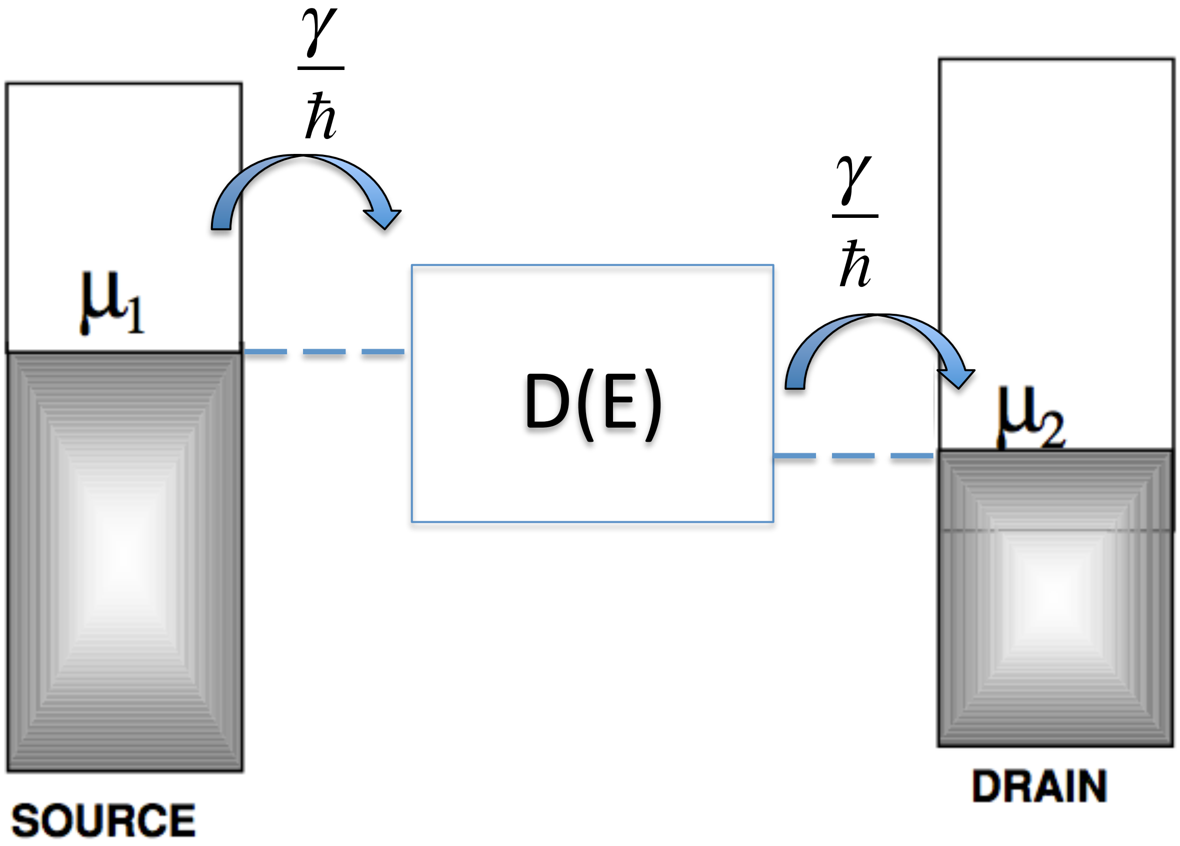
Number of Electrons (Equilibrium)



$$N = \int_{-\infty}^{\infty} dE \cdot n(E) = 2\int_{-\infty}^{\infty} dE \cdot D(E) \cdot f(E)$$

This is for equilibrium and
2 is multiplied for spin

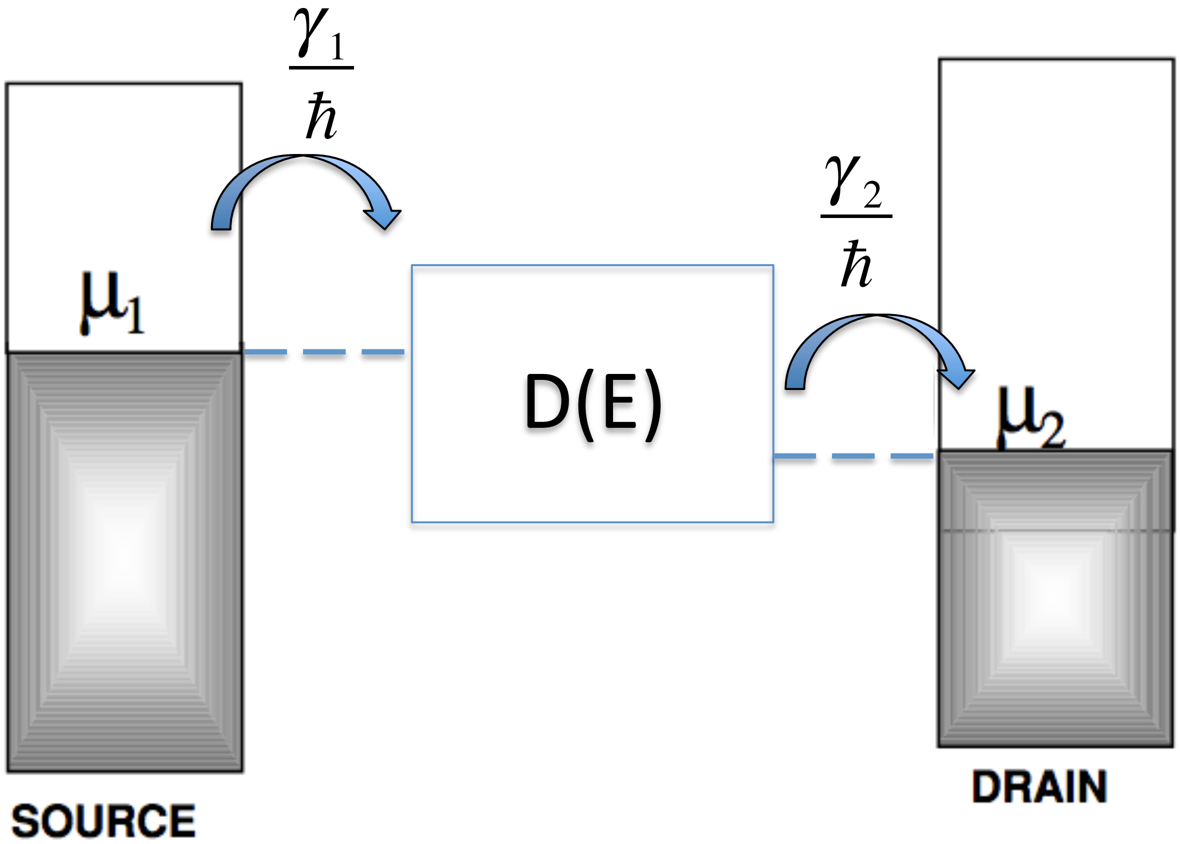
Number of Electrons (Non-Equilibrium)



$$\underbrace{N = 2\int_{-\infty}^{\infty} dE \cdot D(E) \cdot \left[\frac{f_1(E)+f_2(E)}{2}\right]}_{f_{channel}(E)}$$

This is for non-equilibrium and here we assume escape rate γ is same for both sides.

Asymmetric Device

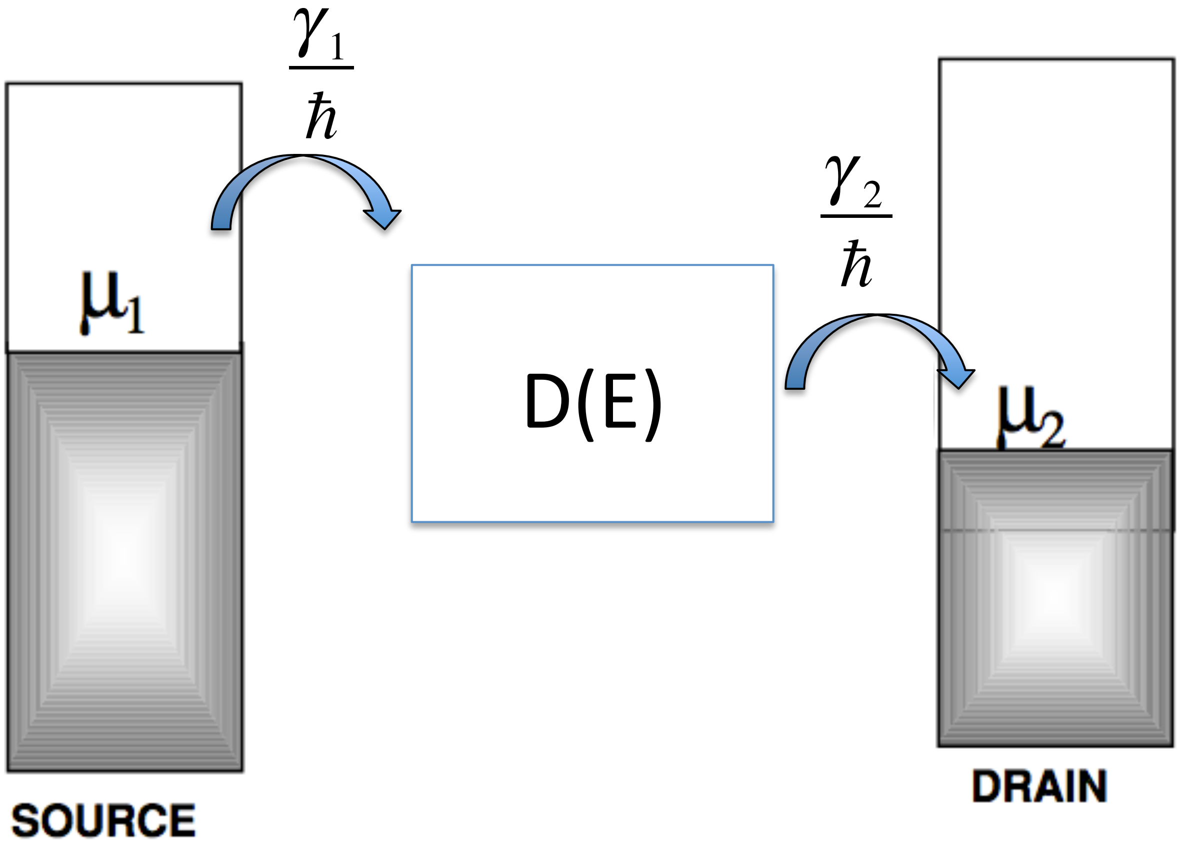


$$\boxed{N = 2\int_{-\infty}^{\infty} dE \cdot D(E)\underbrace{\left[\frac{\gamma_1 f_1(E)+\gamma_2 f_2(E)}{\gamma_1+\gamma_2}\right]}_{f_{channel}(E)}}$$

This is for non-equilibrium and here we assume escape rate γ is different for both sides (γ_1 & γ_2)

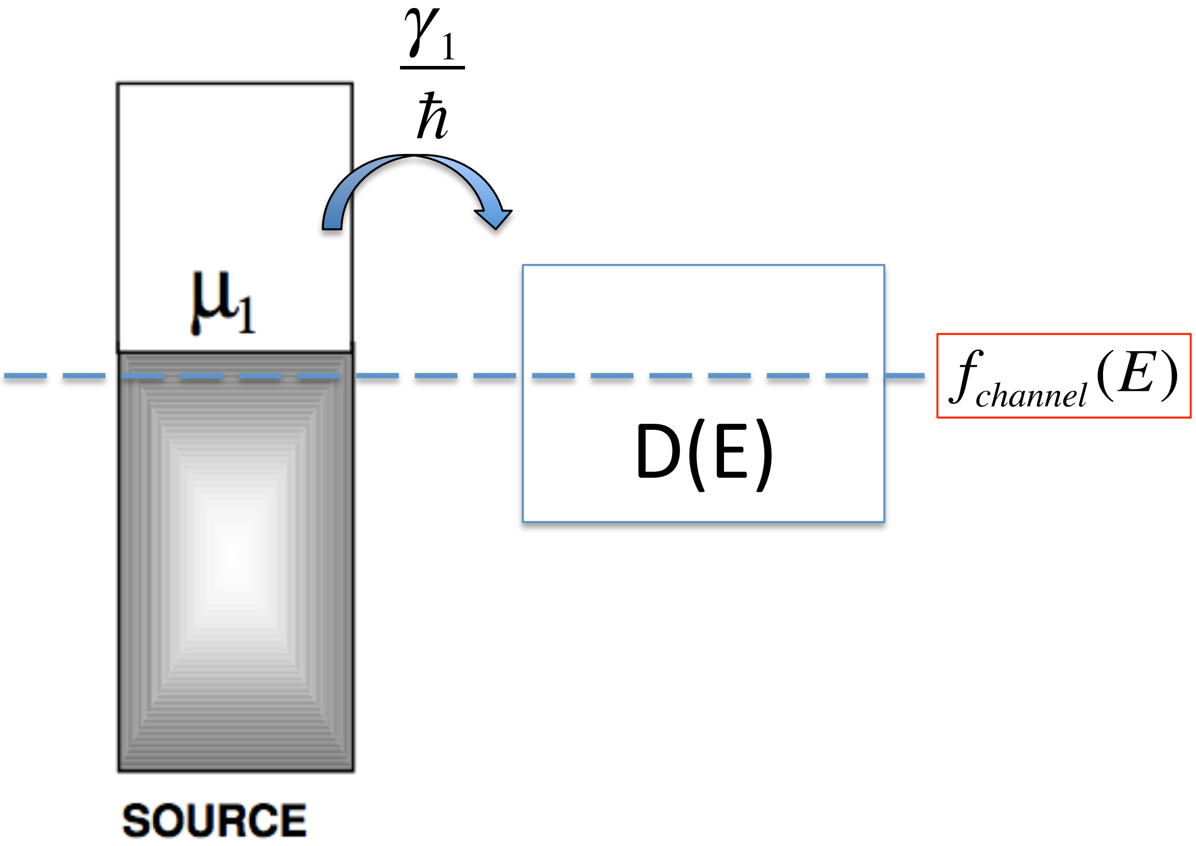
ASSYMETRIC COUPLING & CURRENT

Current in Asymmetric Device



$$I = \frac{q}{h}\int_{-\infty}^{\infty} dE \cdot D(E)\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\left[f_1(E) - f_2(E)\right]$$

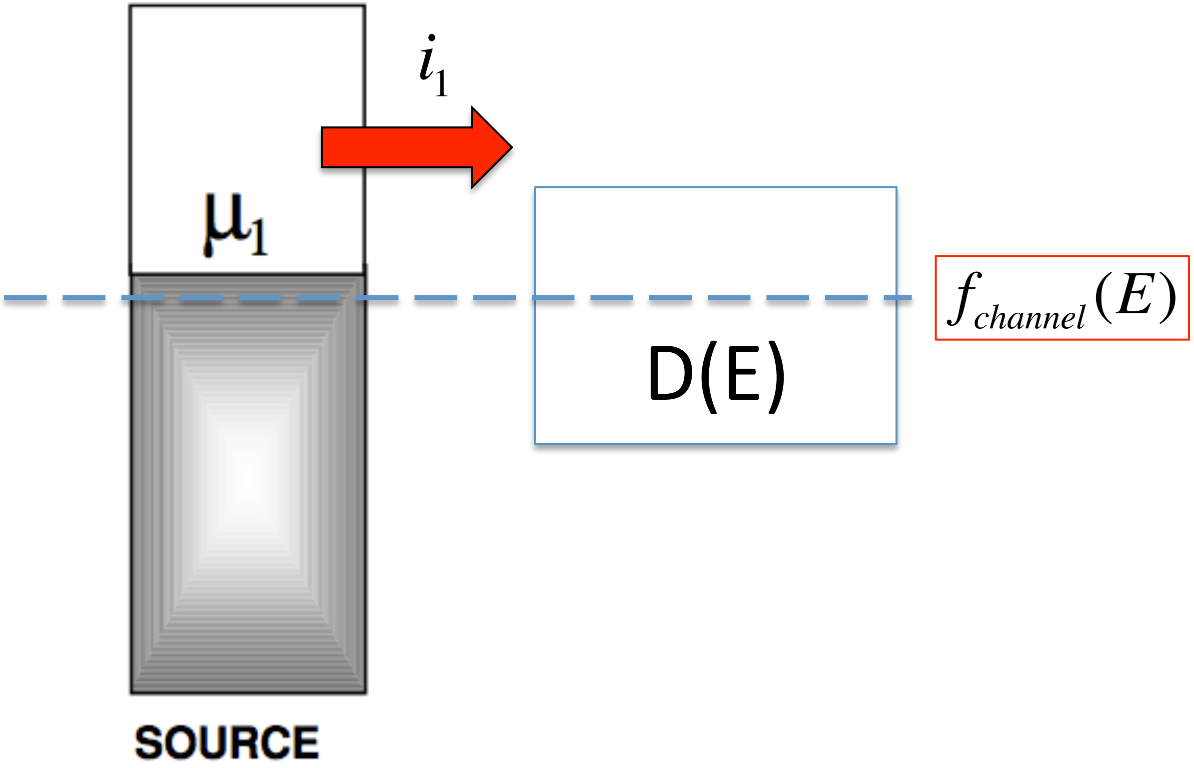
Current in Asymmetric Device



$$i_1^{in} = \frac{q\gamma_1}{\hbar} dE \cdot D(E) \cdot f_1(E)$$

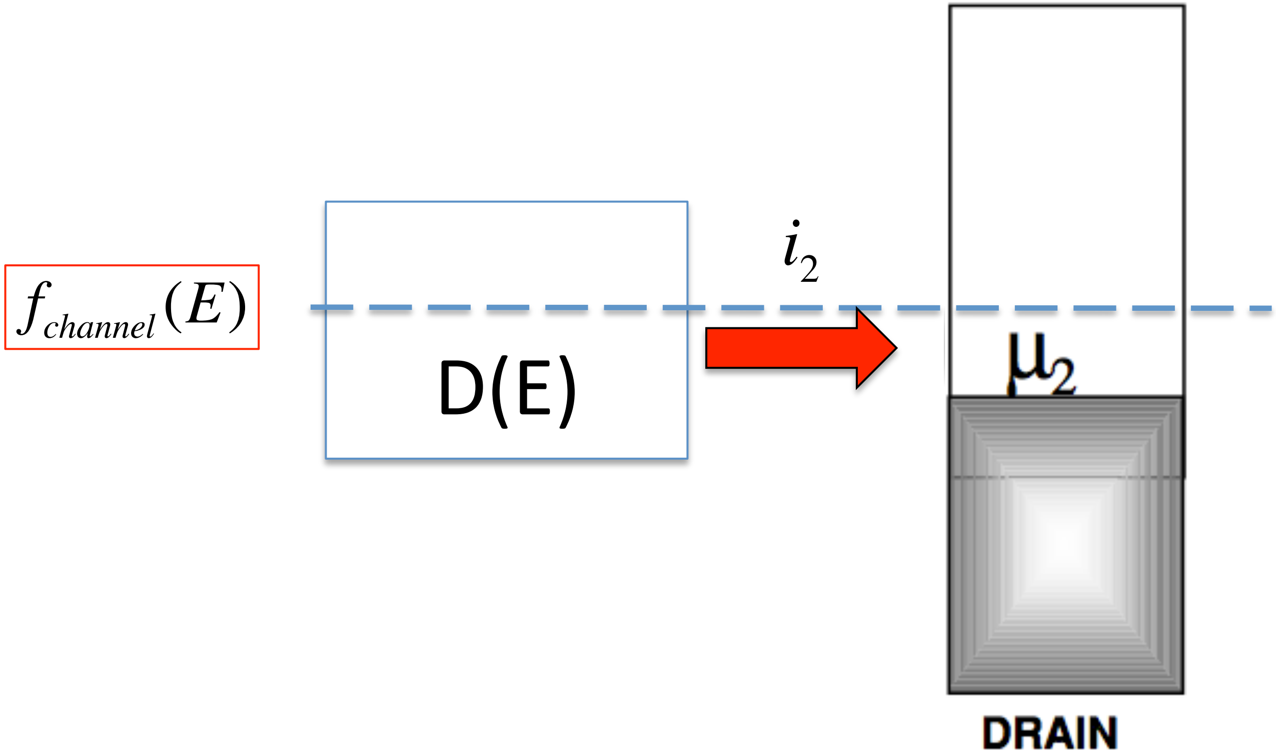
$$i_1^{out} = \frac{q\gamma_1}{\hbar} dE \cdot D(E) \cdot f_{channel}(E)$$

Current in Asymmetric Device



$$\boxed{i_1 = i_1^{in} - i_1^{out} = \frac{q\gamma_1}{\hbar} dE \cdot D(E) \cdot \left[f_1(E) - f_{channel}(E) \right]}$$

Current in Asymmetric Device



$$\boxed{i_2 = i_2^{out} - i_2^{in} = \frac{q\gamma_2}{\hbar} dE \cdot D(E) \cdot \left[f_{channel}(E) - f_2(E) \right]}$$

Charge Conservation

$$i_1 = i_1^{in} - i_1^{out} = \frac{q\gamma_1}{\hbar} dE \cdot D(E) \cdot \left[f_1(E) - f_{channel}(E) \right]$$

$$i_2 = i_2^{out} - i_2^{in} = \frac{q\gamma_2}{\hbar} dE \cdot D(E) \cdot \left[f_{channel}(E) - f_2(E) \right]$$

$$\boxed{i_1 = i_2}$$

Charge Conservation

$$\boxed{\gamma_1 \cdot \left[f_1(E) - f_{channel}(E) \right] = \gamma_2 \cdot \left[f_{channel}(E) - f_2(E) \right]}$$

$$\boxed{f_{channel}(E) = \frac{\gamma_1 \cdot f_1(E) + \gamma_2 \cdot f_2(E)}{\gamma_1 + \gamma_2}}$$

Weighted Average

Channel Fermi Energy?

Weighted Average

$$\boxed{f_{channel}(E) = \frac{\gamma_1 \cdot f_1(E) + \gamma_2 \cdot f_2(E)}{\gamma_1 + \gamma_2}}$$

$$f_1(E) \equiv \frac{1}{1 + \exp[(E - \mu_1)/k_{\mathrm{B}}T]}$$

$$f_2(E) \equiv \frac{1}{1 + \exp[(E - \mu_2)/k_{\mathrm{B}}T]}$$

there is no $\mu_{channel}$

$$\boxed{f_{channel}(E) \neq \frac{1}{1 + \exp\left[\left(E - \mu_{channel}\right)/k_B T\right]}}$$

Charge Conservation

$$i_i = \frac{q\gamma_1}{\hbar} dE \cdot D(E) \cdot \left[f_1(E) - \frac{\gamma_1 \cdot f_1(E) + \gamma_2 \cdot f_2(E)}{\gamma_1 + \gamma_2} \right]$$

$$\boxed{i_i = \frac{q}{\hbar} dE \cdot D(E) \cdot \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \cdot \left[f_1(E) - f_2(E) \right]}$$

$$\Downarrow$$

Energy Integral

$$\boxed{I = \frac{q}{\hbar} \int dE \cdot D(E) \cdot \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \cdot \left[f_1(E) - f_2(E) \right]}$$

SELF CONSISTENT CURRENT CALCULATION

Charge Conservation

$$\boxed{\gamma_1 = \gamma_2}$$

$$\boxed{I = \frac{q}{\hbar}\int dE \cdot D(E-U)\frac{\gamma}{2}\left[f_1(E) - f_2(E)\right]}$$

$$\boxed{U = U_L + U_0\left(N - N_0\right)}$$

$$\boxed{N = \int dE \cdot D(E-U)\left[\frac{f_1(E) + f_2(E)}{2}\right]}$$

$$\boxed{\gamma_1 \neq \gamma_2}$$

$$\boxed{I = \frac{q}{\hbar}\int dE \cdot D(E-U)\frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2}\left[f_1(E) - f_2(E)\right]}$$

$$\boxed{U = U_L + U_0\left(N - N_0\right)}$$

$$\boxed{N = \int dE \cdot D(E-U)\left[\frac{\gamma_1 f_1(E) + \gamma_2 f_2(E)}{\gamma_1 + \gamma_2}\right]}$$

CURRENT ($U_0 \rightarrow 0$)

$$\boxed{I = \frac{q}{\hbar}\int dE \cdot D(E-U)\cdot \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\cdot\left[f_1(E)-f_2(E)\right]} \qquad (I)$$

$$\boxed{N = \int dE \cdot D(E-U)\cdot\left[\frac{\gamma_1\cdot f_1(E)+\gamma_2\cdot f_2(E)}{\gamma_1+\gamma_2}\right]} \qquad (II)$$

$$\boxed{U = U_L + U_0\left(N-N_0\right)} \qquad (III)$$

How to solve this?

If $U_0=0$ => $U=U_L$ then these two equations (II and III) are decoupled

CURRENT ($U_0 > 0$)

$$\boxed{I = \frac{q}{\hbar}\int dE \cdot D(E-U)\cdot \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\cdot\left[f_1(E)-f_2(E)\right]} \qquad (I)$$

$$\boxed{N = \int dE \cdot D(E-U)\cdot\left[\frac{\gamma_1\cdot f_1(E)+\gamma_2\cdot f_2(E)}{\gamma_1+\gamma_2}\right]} \qquad (II)$$

$$\boxed{U = U_L + U_0\left(N - N_0\right)} \qquad (III)$$

How to solve this?

i. Assume an U (by guess),
ii. Find a N using equation (*II*)
iii. Find a new U using equation (*III*)
iv. Do step ii and iii until converge
v. Using equation (*I*) calculate current

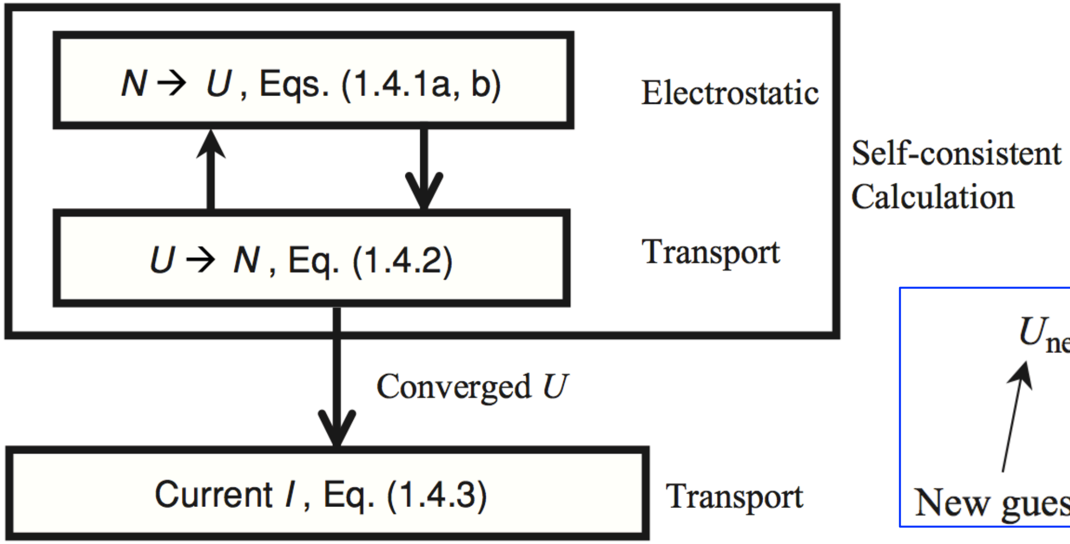
Self Consistent Solution

$$\boxed{I = \frac{q}{\hbar}\int dE \cdot D(E-U)\cdot \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\cdot\left[f_1(E)-f_2(E)\right]} \quad (I)$$

$$\boxed{N = \int dE \cdot n(E) = \int dE \cdot D(E-U)\cdot\left[\frac{\gamma_1\cdot f_1(E)+\gamma_2\cdot f_2(E)}{\gamma_1+\gamma_2}\right]} \quad (II)$$

$$\boxed{U = U_L + U_0\left(N-N_0\right)} \quad (III)$$

How to solve?



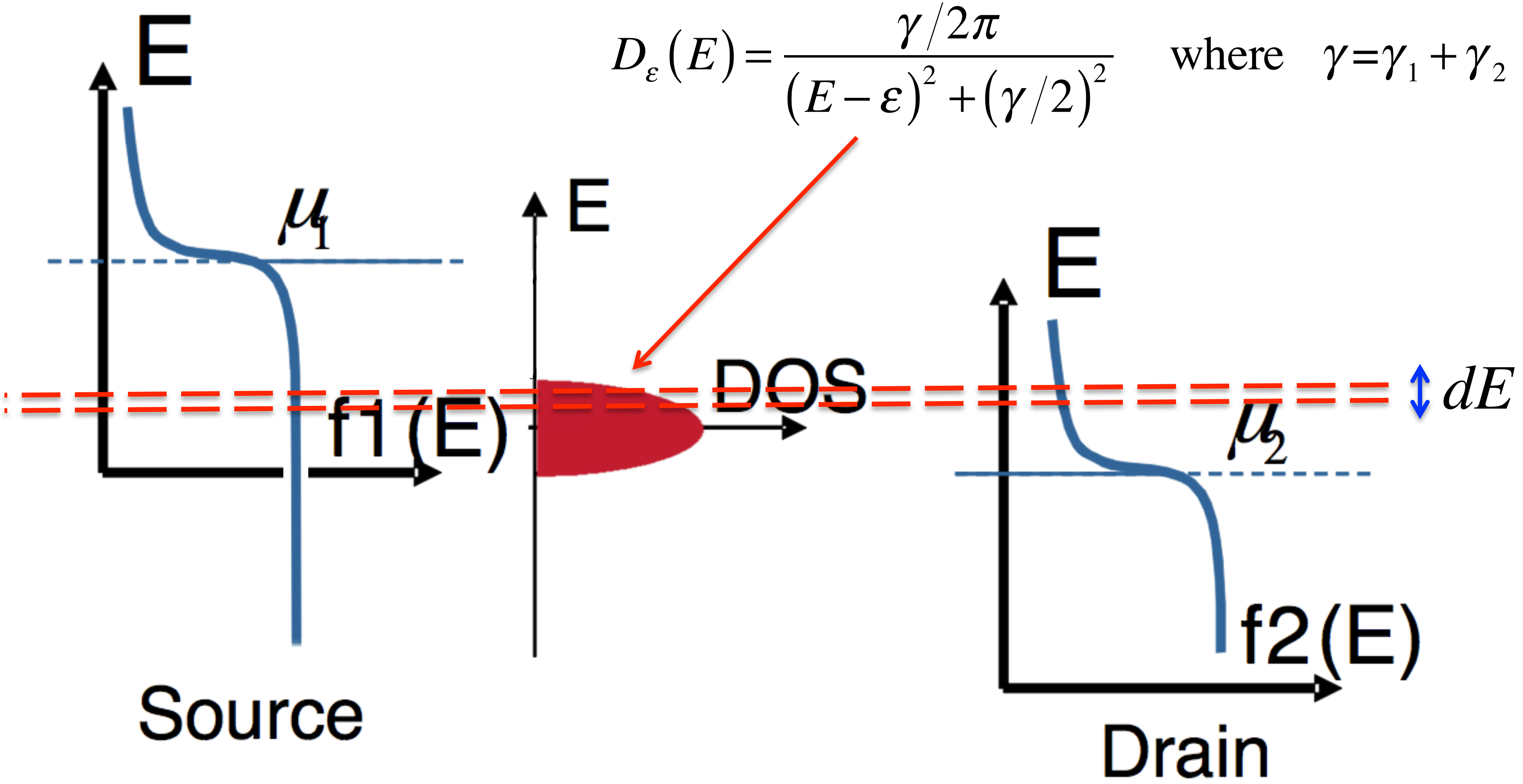
$$\boxed{\underset{\text{New guess}}{\underset{\uparrow}{U_{\text{new}}}} = \underset{\text{Old guess}}{\underset{\uparrow}{U_{\text{old}}}} + \alpha(\underset{\text{Calculated}}{\underset{\uparrow}{U_{\text{calc}}}} - U_{\text{old}})}$$

Self Consistent Solution (critical condition fraction of kT)

CURRENT THROUGH A SINGLE LEVEL REVISITED!!!

CURRENT THROUGH SINGLE ENERGY LEVEL

$$D_\varepsilon(E) = \frac{\gamma/2\pi}{(E-\varepsilon)^2 + (\gamma/2)^2} \quad \text{where} \quad \gamma = \gamma_1 + \gamma_2$$



Current : $$I = \frac{q}{\hbar}\int dE \cdot D_\varepsilon(E) \cdot \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2} \cdot \left[f_1(E) - f_2(E)\right] = \frac{q^2}{h}$$

CURRENT THROUGH SINGLE ENERGY LEVEL

Current :

$$I = \frac{q}{\hbar} \int_{-\infty}^{+\infty} \mathrm{d}E\, D_{\varepsilon}(E) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1(E) - f_2(E)]$$

At low temperature:

$$\begin{aligned} f_1(E) - f_2(E) &= 1 \quad \text{if} \quad \mu_1 > E > \mu_2 \\ &= 0 \quad \text{otherwise} \end{aligned}$$

$$I = \frac{q}{\hbar} \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \int_{\mu_2}^{\mu_1} \mathrm{d}E\, D_{\varepsilon}(E) \quad = \frac{q}{\hbar} \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} (\mu_1 - \mu_2) \frac{(\gamma_1 + \gamma_2)/2\pi}{(\mu - \varepsilon)^2 + (\gamma_1 + \gamma_2)^2}$$

$$\mu_1 - \mu_2 = qV_{\mathrm{D}}$$

$$G \equiv \frac{I}{V_{\mathrm{D}}} = \frac{q^2}{h} \frac{4\gamma_1 \gamma_2}{(\gamma_1 + \gamma_2)^2} = \frac{q^2}{h} \qquad \gamma_1 = \gamma_2$$